AGRODEP
African Growth & Development Policy
modeling consortium
FACILITATED BY IFPRI

AGRODEP Technical Note 11

March 2015

Macro Econometric Modelling: A Practical Approach under EViews, with a Focus on Africa

Jean Louis Brillet

Table of Contents

1. Introduction 6

2. A short history of macroeconometric modelling 6

2.1 The First Modelling Efforts: Tinbergen 6

2.2 Developing the First Operational Models: The Cowles Commission (Later Cowles Foundation) 6

2.3 The Klein Models 7

2.3.1 The Klein-Goldberger Model 7

2.3.2 The Brookings Model 9

2.3.3 The Wharton Model 9

2.3.4 Other Models 9

2.4 The Fall from Favor 10

2.4.1 The Oil Shock 10

2.4.2 The Lucas Critique and DSGEs 10

2.4.3 The Sims Critique and VAR Models 11

2.5 Model Characteristics 12

2.5.1 Comparing the Quality - Calibration 12

2.5.2 The Problem is the Formula 12

2.5.3 SEMs also Use Optimization 13

2.5.4 Rational Expectations? 13

2.5.5 Comparing Structural Models and VARs 14

2.5.6 Quasi Accounting Models 15

2.5.7 Conclusion 15

3. Application of Modeling: South Africa 18

3.1 Building a Model 18

3.1.1 Preparing the Model - First Steps 18

3.1.2 Estimation 19

3.1.3 Solving and Testing over the Past 19

3.1.4 Solving and Testing over the Future 19

3.1.5 Using Model for Forecasts and Policy Studies 19

3.1.6 How to organize the development of the model 20

3.2 First Example: South Africa 23

3.2.1 The Economic Formulations 24

3.2.2 Defining the Model Equations 41

3.3 The Second Task: Obtaining the Data 49

3.3.1 The Data Needed by the Model 49
3.3.2 The South African Case 53
3.4 The Third Task: Estimating the Equations 56
3.4.1 Stationarity and the Dickey-Fuller Test 56
3.4.2 The Production Block 62
3.4.3 Prices 83
3.4.4 Household consumption 90
3.4.5 Exports 92
3.4.6 Conclusion 95
3.5 The Fourth Task: Simulating the Model 95
3.5.1 A First Test: Checking the Residuals In The Identities 96
3.5.2 Simulating the Model over the Past 97
3.5.3 Testing the Model over the Future 99
3.5.4 The Results 101
3.5.5 Stochastic Simulations 129
3.6 Rational Expectations 137
3.6.1 The Framework 137
3.6.2 Consequences for Model Simulations 137
3.6.3 Our Example 137
3.6.4 The Tests 138
4 Another Country: Sénégal 144
4.1 Reading the Data: Sen_Read.Prg 144
4.2 Generating the Model Series: Sen_Genr.Prg 144
4.3 Building the Model Framework and Behaviors: Sen_Model.Prg 144
4.3.1 Capacity 144
4.3.2 Productive Investment 146
4.3.3 Labor Productivity Trend 147
4.3.4 Employment 148
4.3.5 Unemployment 149
4.3.6 Wage Rate 151
4.3.7 Imports Deflator 152
4.3.8 Exports Deflator 153
4.3.9 Household Consumption 155
4.3.10 Imports At Constant Prices 156
4.3.11 Exports At Constant Prices 157

4.4 A Forecast *158*
4.5 The Shocks *161*
4.5.1 An Increase Government Demand 162
4.5.2 An Increase in VAT 164
4.5.3 An Increase in Quotas Applied to Local Exports 166
4.5.4 An Increase in Quotas Applied to Local Imports 168
4.5.5 A Decrease in the Local Tariffs Rate 170
4.5.6 A Devaluation 172
5. Bibliography 173
Econometrics and Statistics *173*
Macroeconomics *175*
Models and Modelling *176*

1. Introduction

Let us start with a definition of the field of macroeconometric modelling by one of the best experts, whose recent demise was deeply felt by all who knew him:

"A schematic simplification that strips away the non-essential aspects to reveal the inner working, shapes, or design of a more complicated mechanism" (Lawrence Klein, 1983: 1).

So many books have been written on the subject of macroeconometric modelling that the usefulness of adding a new general contribution can be questioned. Our approach will be original in two ways.

First, it will be extremely developed, with the goal of giving modelers a complete set of elements which will allow them to build their own models. To this end, our presentation is complemented by examples using the EViews software, from general strategies to technical details. The associated programs and files will be provided.

Second, it will focus on Africa, to which all our examples will relate. We will also present the specific issues, problems and strategies for modeling this region and developing countries in general.

2. A short history of macroeconometric modelling

In this chapter we present the history of macroeconomic modelling, and the options available to present modelers.

2.1 The First Modelling Efforts: Tinbergen

Jan Tinbergen (1903–1994) was the founder of economic modelling. Before him, global macroeconomics was mostly a literary science, with no full formalization. In the thirties, Tinbergen introduced

- building a full system linking formalized behaviors and identities,
- the separation of model elements into exogenous and endogenous,
- formalized tools for economic policy through the establishment of links between the exogenous and the policy targets, in equal number (the “Tinbergen rule”), and
- econometrics in models.

This allowed him to design a series of national models, first for the Netherlands (1936) and then the United States (1939).

2.2 Developing the First Operational Models: The Cowles Commission (Later Cowles Foundation)

World War II put a quasi-stop to the development of economic theory, but the reconstruction policies following the end of the war called for quantifying tools. The main effort to this end was made by the

Cowles Commission, which was founded in 1932 by Alfred Cowles but became fully efficient in 1943 in Chicago under Jacob Marschak. It brought together an impressive list of economists, among them Kenneth Arrow, Gérard Debreu, James Tobin, Franco Modigliani, Herbert A. Simon, Lawrence Klein, Trygve Haavelmo, Leonid Hurwicz, and Harry Markowitz. Christ (1994) says the commission called for the following:

- Systems of simultaneous economic behaviors
  - using observable variables without error
  - using discrete variables.
- A priori determination of exogeneity and endogeneity.
- A priori identified structural equations, following economic theory.
- A dynamically stable system of equations, with a reduced form.
- Using linear or logarithmic equations and disturbances.
- Independence of the assumptions.
- Normally distributed disturbances with zero means, finite and constant covariance.
- A nonsingular covariance matrix, and serial independence.
- In principle, simultaneous estimations.

The earliest economic models did not follow all of these principles, yet all are still valid today for the family of macroeconometric structural models.

2.3 The Klein Models

2.3.1 The Klein-Goldberger Model

The Klein-Golberger model was the first instance of an operational model. While it included most of the elements of present modeling products, it lacked many elements present in standard current versions (Appendix I). It did not formalize productive capacity, exports were exogenous, unemployment had no role, and the model did not follow an error-correction framework.

Table 1. Variables of the K-G model

Endogenous Variables	Exogenous Variables
C Consumption	F_R Farm exports
D Depreciation	G Government expenditures + exports
F_I Imports	h Hours of work
I Investment	L_B Percentage excess reserves
i_L Long-term interest rate	N_E Entrepreneurs
i_S Short-term interest rate	N_G Government employees
K Capital stock	N_L Labor force
L_1 Household liquid assets	N_P Population
L_2 Business liquid assets	P_F Import price level
M National income	R_2 Farm subsidies
N_W Employees	t Time trend
P Nonwage nonfarm income	T_C Corporate taxes
P_C Corporate profits	T_E Indirect taxes
p Price level	T_N Nonwage nonfarm non-corporate taxes less transfers
p_R Farm price level	T_R Farm taxes less transfers
Q Gross National Product	T_W Wage taxes less transfers
R_1 Farm income	W_2 Government wage bill
S_B Corporate surplus	
S_C Corporate saving	
w Wage rate	
W_1 Private wage bill	

The equations in the Klein Goldberger model

1. $C = -22.26 + .55(W_1 + W_2 - T_W) + .41(P - T_C - T_N - S_C) + .34(R_1 + R_2 - T_R) + .26C_{-1} + .072(L_1)_{-1} + .26N_P$

2. $I = -16.71 + .78(P - T_C - T_N + R_1 + R_2 - T_R + D)_{-1} - .073K_{-1} + .14(L_2)_{-1}$

3. $S_C = -3.53 + .72(P_C - T_C) + .076(P_C - T_C - S_C)_{-1} - .028(S_B)_{-1}$

4. $P_C = -7.60 + .68P$

5. $D = 7.25 + .10 \dfrac{K + K_{-1}}{2} + .044(Q - W_2)$

6. $W_1 = -1.40 + .24(Q - W_2) + .24(Q - W_2)_{-1} + .29t$

7. $Q - W_2 = -26.08 + 2.17[h(N_W - N_G) + N_E] + .16 \dfrac{K + K_{-1}}{2} + 2.05t$

8. $w - w_{-1} = 4.11 - .74(N_L - N_W - N_E) + .52(p_{-1} - p_{-2}) + .54t.$

9. $F_I = .32 + .006(M - T_W - T_C - T_N - T_R) \dfrac{p}{p_F} + .81(F_I)_{-1}$

10. $R_1\,(p/p_R) = -.36 + .054(W_1 + W_2 - T_W + P - T_C - T_N - S_C)\,(p/p_R) - .007[(W_1 + W_2 - T_W + P - T_C - T_N - S_C)\,(p/p_R)]_{-1} + .012F_R.$

11. $p_R = -131.17 + 2.32p$

12. $L_1 = .14(M - T_W - T_C - T_N - S_C - T_R) + 76.03(i_L - 2.0)^{-.84}$

13. $L_2 = -.34 + .26W_1 - 1.02i_S - .26(p - p_{-1}) + .61(L_2)_{-1}$

14. $i_L = 2.58 + .44(i_S)_{-3} + .26(i_S)_{-5}$

15. $100\,\dfrac{i_S - (i_S)_{-1}}{i_S} = 11.17 - .67L_B$

16. $K - K_{-1} = I - D.$

17. $S_B - (S_B)_{-1} = S_C$

18. $W_1 + W_2 + P + R_1 + R_2 = M$

19. $C + I + G - F_I = M + T_E + D$

20. $h(w/p)N_W = W_1 + W_2$

21. $Q = M + T_E + D$

2.3.2 The Brookings Model

In the early 1960s Klein became the leader of the "Brookings-SSRC Project," established to construct a detailed short-term model of the U.S. economy. It gave birth in 1965 to the Brookings Quarterly Econometric Model of the United States. The model's originality was its detail, its short term (quarterly) periodicity, and some theoretical improvements.

2.3.3 The Wharton Model

When Lawrence Klein moved to the University of Pennsylvania, he founded the Wharton Econometric Forecasting Associates. Around 1966, he constructed the Wharton Econometric Forecasting Model. This model was considerably smaller than the Brookings. It obtained a good reputation as a policy tool.

2.3.4 Other Models

Many US institutions, particularly universities, developed their own models. The most notable is perhaps the US Fair Model, developed by Ray C. Fair at Yale University, with 130 equations (30 of which estimated). Its originality lies in its availability—for free, including the associated software-- and also its longevity. A world model is also available, including 38 countries.

Although the US was and still is a leader in terms of modelling projects, models were built in many countries. In France, for example, one can cite:

- The Dymamic Multi Sectoral model (1976) and the METRIC model developed by the National Institute for Economic Studies
- The MEFISTO model of the Bank of France (1992).
- The MESANGE (2002) and MZE (Modèle Zone Euro, 2003) models of the Ministry of Finance and INSEE.

Developing countries produced models, too, although they faced data issues. (A detailed list of African models to follow.)

2.4 *The Fall from Favor*

In the seventies, the science of structural econometric models and economic theory in the arena of macroeconomics had globally stabilized. Acceptability of this type of model for both forecasts and policy analysis was at its peak, and the stability of economic growth at the time made for an easier task.

But soon two events occurred: the 1973 oil shock, and the Lucas Critique.

2.4.1 The Oil Shock

The large shock that the developed countries experienced in 1974 highlighted how difficult it was to forecast economic variables. Moreover in the seventies, the GDP started fluctuating much more than before, making the forecasting task more difficult. Not only were models unable to forecast the shock, but they also faced a more difficult task, with subsequent lower efficiency.

2.4.2 The Lucas Critique and DSGEs

The critique was formulated by Robert Lucas in a 1976 paper. In his own words:

"Given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models."

This means, in other words, that behavioral equations estimated using present and lagged explanations are associated to an economic framework which might not be applicable in the future.

This was completed in 1977 by Kydland and Prescott, who published "Rules Rather than Discretion: The Inconsistency of Optimal Plans". They state in particular that agents, being rational --an assumption absent from Lucas' paper-- they can forecast the changes of policy and act accordingly. Any policy conducted in the short run and announced to be permanent but bound to be changed in the future (maybe because of its cost) will be inefficient as the response of agents will not follow it.

To solve this problem in a modelling context, economists have supposed:

- That the agents are able to forecast the changes in policy rules (including the changes in the size of variables),
- That they are able to forecast the consequences for the economic equilibrium, and
- That the agents use this knowledge to maximize their utility over time.
- The new model can either:
- Use traditional equations (including estimations) using future variables as part of the explanation,
- Use equations coming from a maximization process (and future elements), and
- Maximize the utility inside the program.

This new framework led to the development of a new class of tools, the first Computable General Equilibrium (CGE) models: the Dynamic Stochastic General Equilibrium models. Their main characteristic is overcoming the Lucas critique by supposing that agents are able to build up rational expectations of the future, and maximize their utility accordingly.

In contrast to Structural Econometric Models, they:

- use formalized rational expectations.
- are general calibrated, even though estimation can be present, either by applying econometrics to the formulas, or using value coming from estimated models.
- apply intertemporal optimization, either through a special algorithm or the use of formalized derivatives.

2.4.3 The Sims Critique and VAR Models

In 1980, Christopher Sims made a different critique of structural models. He argued that the restriction imposed by SEMs to the formulations are arbitrary-- "the identification claimed for existing large-scale models is incredible"-- and that it was much better to let econometrics decide on the formulations, introducing a large number of explanatory variables in a possibly complex lagged structure called a Vector Auto Regressive mode, or VAR.

Typically, each variable in a VAR model has an equation explaining its evolution based on its own lags and the lags of the other model variables. Obviously, this calls for much less theory than SEMs or even DSGEs. All the equations are backward-looking; Sims was an ardent opponent of rational expectations.

Note that all variables have to be of the same order of integration. The following cases are distinct:

All the variables are I(0) (stationary): one is in the standard case, i.e. a VAR in level.

- All the variables are I(d) (non-stationary) with d > 0
- The variables are cointegrated: the error correction term has to be included in the VAR. The model becomes a Vector error correction model (VECM) which can be seen as a restricted VAR.
- The variables are not cointegrated: the variables have first to be differenced d times and one has a VAR in difference.

2.5 Model Characteristics

Although SEMs and DSGEs are generally opposed, they have some common characteristics. Both use explicit theoretical formulations, for example, but DSGEs use rational expectations and are generally calibrated. The most important behaviors in DSGEs result generally from an optimization process (consumption, production function). Most of the time, it uses equations determined outside the model using derivatives of the optimized criterion. In rare cases, the optimization is made inside the model, calling for a specific algorithms unavailable in usual packages (GAMS and DYNARE are exceptions).

This makes the systems more complex and more difficult to understand. It also limits the number of behaviors and the size of the model (the use of rational expectations is also a factor).

DSGEs do not provide a full description of the economic equilibrium, in particular of the full budget. This could actually be done, but not done in practice.

2.5.1 Comparing the Quality - Calibration

Calibration needs to choose the values.

It is not always used in DSGEs as coefficients can be obtained by econometrics, sometimes applied outside the model. In any case, the calibrated coefficients have to conform to accepted values in the determination of which econometrics may have played a role.

Actually the problem could very well be reversed: One can start from a calibrated SEM (closer to a DSGE) and use econometrics to get a more precise value, in case of success (and if it conforms to theory, like a DSGE).

2.5.2 The Problem is the Formula

The critique applies to a change on regime, not to the size of assumptions (and the resulting equilibrium), unless they take the explained elements outside the validity interval, or the shape of the formula is no longer adapted.

For instance, it is clear that the assumption of a constant elasticity is not only an approximation, but it becomes less and less acceptable as the elements move to abnormal values.

2.5.3 SEMs also Use Optimization

Most SEMs also use optimization. In our small model of the French economy, this is true for:

- production factors (capital and employment targets minimize the cost of production),
- the wage rate (using a ws – ps formula),
- the value added deflator (with a trade-off between output and margins),
- the trade prices (again with a tradeoff between exports and margins), and
- household consumption (with a tradeoff between present and future, based on the interest rate and forecasted unemployment).

The only non-maximizing behaviors relate to trade at constant prices, unemployment and the change in inventories, although this could be disputed (and an optimization process designed, at least for the last two).

SEM equations are estimated separately, at least in most cases (not for the factors in the small French model), but:

- cointegration allows to separate the behaviors.
- simultaneous estimations can be introduced if desirable.
- DSGEs use calibrated values. Is it better?

2.5.4 Rational Expectations?

Assuming agents are rational is questionable, and that they know (or even can find the solution to) the future model even is even more so. This assumption is necessary to take into account the critique in practice, just as a large enough sample is necessary to apply modern economic techniques. If not, pertinent models cannot be built at all.

There are only imperfect solutions, of which rational expectations is only one, the other one being backward looking models, perhaps formalizing the future based on present and past information. So RE is an ad hoc assumption to allow building of models?

This is less and less true. Actually, in 1976 solving such an operational model was impossible. At present even a several hundred equations model can be solved over a number of periods (it takes some time, though). The problems lie more in reaching an acceptable solution, as the last period solved obviously depends on non-forecasted information.

Backward looking models are still interesting:

- They are much easier to formulate.
- They are much easier to solve.
- They do not need the “rational” assumption.
- They can use expectations, as long as they are defined as a function of present and past elements.

In particular, one can introduce constraints in a BW looking model (e.g. that if a threshold is reached the regime will change). Regime changes are generally gradual, so backward looking models can still be useful in the short term. In the long term, the equilibria are enforced by any model.

2.5.5 Comparing Structural Models and VARs

Here the comparison is much easier, as the differences are clearer.

VARS have little economic logic, and do not conform to formalized theories.

This is both an advantage (consider the Sims critique) and a drawback as we lose an element of control, and the information on the causal channels associated to policies (or shocks on assumptions in general). If the goal was to interpret the economic mechanisms in terms of individual elements, we get absolutely no information.

Of course, you have to believe in the theory, and econometrics is not so helpful here, as it is true that alternate theoretical choices more or less different (and maybe actually opposed) could obtain equivalent or better criteria.

In the short run, VARs might behave better.

It is clear that VARs should behave better on the recent past, as they apply no restrictions. If the image obtained is reliable enough (even in the absence of economic explanation) and if the structure of the economy does not change too much, the image could still be used for the next few periods.

Of course, this more probable for short periodicities, as more information is available (more lags can be introduced) and the number of forecasted periods grows for the same horizon.

In the long run too?

One can argue that the absence of theory makes the VARs less defendant to changes in policy and the economic framework. This is balanced however by the absence of long run formalized equilibriums, present in the last generation of SEMs. This is also true for balances.

And what about shocks?

In principle, the absence of formalized links should limit the use of VARs to forecasts. This is not necessarily true, as even a "black box" structure can provide acceptable responses, if the unidentified causal structure is actually accurate enough.

2.5.6 Quasi Accounting Models

This is actually a particular kind of a SEM. Their main originality is the absence of econometrics.

However, the "behavioral" equations can very well contain theoretical features, depending on the will of the modeler, using calibrated coefficients (with the possible help of SEMs).

Their main advantages are:

- the limited requests in terms of data. The number of periods available can be very small, actually one plus the maximum lag. (Of course it is better to have more, if only to check the validity of the equations and model.)
- the possibility to go into a fine detail. With a highly decomposed model, it is unrealistic to expect econometrics to work (with a higher probability of failure and probably a decrease in the quality of data).

2.5.7 Conclusion

For a potential model builder, the main questions are:

- What goal is my model going to pursue?
- Which models can I build using the available information (data in particular)?
- Which models will best allow me to reach my goal, even partially and with a relative reliability?

The answers to these question will inform the modeler's choice.

Let us suppose the goal is to build a model which allows consistent predictions (even accepting a large margin of error) and assessing the consequence of external or domestic shocks (again focusing on the profiles and mechanisms rather than the actual numerical results). The user can be either a government agency, an academic institution, or even an individual researcher.

For a developed country like France, the options are quite numerous, and most of them have actually been applied. One can find numerous examples of SEMs, VARs and DSGEs developed by all of the above agents.

VARs represent a marginal option. The do allow short term forecasts, but if shocks can be produced, their interpretation and reliability is questionable at best.

DSGEs look better. If one believes in rational expectations, they will probably be the best option, even if structural models can very well be adapted by introducing forward variables in standard behavioral equations, as we shall see. On the other hand, DSGEs they are rather complex, validation is more difficult, and they never provide a full description of national accounts tables, a requisite for an operational use.

Without rational expectations, SEMs are the clear preference. The Lucas Critique still applies, of course, but SEMs remain the only option. This option is flawed indeed, but the teachings of the model remain interesting. In the short run, the rules might be decided as stable or evolving slowly, and in the long run, any prediction is at best indicative, even using DSGEs. One just has to consider the forecast as "all rules being stable."

For example, a government demand shock cannot be pursued forever, as it will lead to unacceptable stocks of debt. To take this into account, one can:

- use a DSGE in which a rule on the stock of debt is introduced.
- use a SEM with rational expectations, with a rule saying the government will stop spending over a certain threshold of debt.
- use a backward looking SEM, and observe what happens if the shock is pursued indefinitely (an unrealistic simulation which will still give interesting information, as the diagnosis will not be affected by the decrease in the shock).
- use a backward looking SEM, but stop the shock after a number of periods. The results will be realistic if we suppose the agents are only backward looking.
- use a backward looking SEM, but formalize the expectations of agents using backward elements. For instance, one can decide that as government debt gets higher, expenditures will decrease according to a certain progression. In a way, this means that the rule will change progressively with the evolution of its determinants.

In the case of a developing country with a relatively short statistical history (in particular of national accounts), the choice is much more restricted, due to several factors.

The first problem is data. There are two issues: the length and the scope.

The first decides if econometrics can be used. The length of series can go from one period (when the system of National accounts has just been implemented) to a large number (for Algeria annual accounts are available since 1978).

In our opinion, yearly estimation can start at 10 periods, where all one can hope for is an indication on the validity of the formula (and the coefficients values one contemplates). But it is only with 20 that one can really consider using econometrics as a validation tool. Of course using quarterly data will generally provide more observations, but the information coming from four quarters is not equivalent to four years.

The second problem has several facets. Essential series can be completely lacking:

- The production factors, i.e. labor and capital. These elements do not appear in the National Accounts, which means that the producers are not the same. For the second, surveys do not give the value and specific methods have to be applied, not necessarily very reliable.
- Some detailed budget elements, like subsidies and taxes.
- The global revenue of agents, in particular households.
- The foreign assumptions (demand addressed to the country and foreign prices). This calls for the knowledge of the share of partner countries in exports and imports, and elements from the countries themselves (imports and production prices).
- Goods and services can be known only at current prices, or the deflators available can be limited.

In all these cases solutions can be found, to a variable cost in terms of model reliability. Additional problems are:

- The absence of sectoral detail, which can be crucial for economies depending on a specific good (like oil producing countries).
- The existence of an important informal sector (already present in developing countries, consider Italy).
- The limited length of some series.
- The quality of the measurements.
- The specific framework

Even with an adequate set of accurate information, modelling a developing country faces specific problems. The economic system is far from stabilized, which means that the evolutions are faster, calling not only for a specific control of convergence processes, but probably some specific formulations, including time-varying coefficients or even equations. For instance, control of prices can be loosened with time, or non-productive employment can be abandoned. This is particularly true for the framework of monetary mechanisms, which can change a lot during the process.

Very often, the economic production is based on specific goods such as oil and gas, ores, or agricultural goods. They have to be identified for the model to have realistic properties. This is true even if the elements are defined as exogenous: mixing exogenous elements with others behaving in a standard way makes the definition of the latter (including econometric estimation) quite unreliable.

If the country is quite open, the role of external trade must be carefully formulated and its properties checked. A limited variation in some of the related parameters can have a very strong impact on the properties of the model.

3. Application of Modeling: South Africa

We will now present a series of applications of modelling, related to African countries. We will start with the easiest case, meaning the one for which the model data is available.

We will start by describing the process of building a model, including the techniques used for bypassing the difficulties faced (mostly due to the unavailability of the required series) in terms of scope, time length and quality. We will present the finalized model and its properties.

We shall present the results for two countries, in order of difficulty: South Africa and Senegal.

In another document (see the user guide), we will present the tools allowing to perform the tasks described in the first part.

3.1 Building a Model

First, let us give a brief overview of the organization of the model building process.

3.1.1 Preparing the Model - First Steps

The first step in the building of any model is producing a draft which ensures some compatibility between available data (wherever it might come from) and the type of model the builder has in mind (goal, scope, nature of the variables, underlying theory).

Knowing the scope of available data, the builder will define a model framework for which values can be attributed to all variables, either using available elements or by computation. This means that a first decision has to be made as to the field described by the model, the variables used as assumptions, and the variables it shall compute. Moreover, the builder must divide the equations into identities, which set indisputable links between variables, and equations describing the behavior of agents, for which the final formulation will be based on past evolutions of the associated elements.

The first task will be to gather, by reading from files and transforming the data, the full set of variables needed by the model to define the form of the identities and give a first assessment of the behaviors he intends to describe. The builder must check for which periods the necessary data is known, and that on these periods identities hold true. If some elements are not available, one uses the best proxies obtainable; and if this also fails, some imagination is necessary.

He can also make a first economic analysis of the framework implied by model specifications (greatly helped by EViews).

3.1.2 Estimation

In the second phase, one will look for a satisfying description of the behavior of agents, by checking economic theory against available data. The builder shall define alternate formulations with unknown parameters, compute for each formulation the values which give the best explanation of past evolutions, and make his selection using as criteria both statistical tests and compliance to economic theory. This process can call for the introduction of new variables, or changes in some definitions, which will mean reformulating some identities.

3.1.3 Solving and Testing over the Past

Once the full model is defined, one can try to solve it.

- Check the set of equations, data and parameters by applying each formula separately on the sample period. If the estimation residuals have been introduced as additional elements, the process should give the historical values in all cases.
- Simulate the full model on the same period, temporarily setting the residuals to zero. This will show if taking into account current and lagged interactions does not amplify the estimation errors too much.
- Measure the reactions of the equilibrium to a change in assumptions, for instance the exogenous component of demand. Compare the results with the teachings of economic theory and what is known of values given by other models. One should not spend too time here, however, as simulations over the future will provide a much better context.

Discovering discrepancies can lead to changes in some elements of the model, including the set of its variables. This means going back to step 1 or 2.

3.1.4 Solving and Testing over the Future

Once the model has passed all tests on the past, further tests will be conducted, under conditions more representative of its actual use: on the future. For this, one will have to establish values for future assumptions. Again, the sensitivity of the model to shocks will be studied, this time with a longer and smoother base. As to the reliability of baseline results, one shall rely this time on stochastic simulations.

3.1.5 Using Model for Forecasts and Policy Studies

Finally, the model will be considered as fit for economic studies: forecasts and economic policy analysis.

We shall suppose we are using a dedicated package like EViews (even if some people still model through a spreadsheet).

3.1.6 How to organize the development of the model

Let us now consider the organization of the model production task. To create a model, two extreme types of organization can be considered: the methodological option, or improvisation.

With the first method, the model builder:

- specifies a complete, coherent model (including accounting equations), precisely separating assumptions from results,
- looks for the necessary series,
- estimates behavioral equations, and
- uses the subsequent model.

Applying such a framework is obviously unrealistic, as many backtrackings will be necessary in practice:

- Some series will show up as unavailable, and it will be necessary to replace them or to eliminate them from formulations. Thus, in the absence of series for interests paid by firms, one will have to be content with profits before interests.
- Some estimations will give unsatisfactory results: it will be necessary to change formulations, to use additional or alternate series. Thus, a formulation in levels might have to be replaced by a formulation in logarithms (constant elasticities) or in growth rates; one will be led to explain the average monthly wage instead of the hourly wage, and to introduce in this last explanation the evolution of the minimal wage. For an oil producing country, it will be necessary to identify oil (and non-oil products) in both production and exports.
- New ideas will appear during estimation. For example, a recent article on the role of foreign direct investment might lead to test an original formulation.
- Formal errors are going to be identified. Thus, an element (a type of pension) might have been forgotten from households' income.
- Some variables defined as assumptions are going to appear sufficiently influenced by results to see their status modified.

Improvisation's first and probably most important task is preparing the production of the model. This includes:

- the organization of tasks, like producing at first single country models, for a world modelling project;
- economic issues, like choosing the complexity of the production function, or the decomposition of products; and

- technical issues, like the number of letters identifying the country in a world model series names.

One might be tempting to start model production as soon as possible, but it is extremely important to spend enough time at the start evaluating the options and choosing a strategy. Realizing much later that he has chosen the wrong options, the builder is faced with two unattractive solutions: continuing a process leading to a subpar model, or backtracking to the point where the choice was made.

3.1.6.1 Preparing the Model: the Logical Framework

At the start of the model building process, the modeler (or the team) has at least general ideas about the logic of the model he wants to build, and information about the set of available data.

Things can be even more advanced:

- The data can be directly available, almost always as a computer file, but not necessarily in the format needed by the modelling package.
- Equations may have already been established, either as formulas or even estimated items, if the modeling is the continuation of an econometric study.

In any case, the first stage in the process should lead to the following:

- A fully defined set of equations, except for the actual estimated formulas.
- The corresponding set of data.

Obviously these two tasks are linked, as equations are established on the basis of available data and the data is produced to fit the model equations. This suggests that they are normally processed in parallel; however, it is quite possible to produce most of the data before the equations are defined. Some concepts (the supply-demand equilibrium at constant and current prices, employment, interest rates) will certainly appear in the model, but some model-specific variables will have to wait. It is also possible to produce the model specification before any data is available. Of course, writing an identity, or stating the equation to be estimated, does not require data. It is only the application-- checking the identity is consistent, or estimating the equation-- which does. Still, one must be reasonably sure that the data will be available, or that there will be a reasonable technique to estimate it. One can even produce a first version of the program transforming into model concepts the original data, once these concepts are completely defined but before any data is technically available (just their definition).

One can compare the situation with the building of a house. One can draw the plans before the equipment is bought, but its eventual availability (at the right time) must be certain. And the goods can be bought before the plans are completely drawn, but the chance of having to use them must be reasonably high.[1]

[1] As there is a cost to the goods. For free or quasi-free data, the chance can be lowered.

These options are not optimal in the general case, but they can help to gain time. Most modelling projects have a deadline; once the work force is available, the tasks should be processed as soon as possible if one wants to have the best chance of meeting it.

One can question the feasibility of producing a full set of equations before any estimation. What we propose is to replace the future formulations by a "declaration of intent" which states only the variable to be explained and the elements which will explain it. For each equation, the format should be as close as possible to:

Variable = f (list of variables)

The advantages of defining a full model are numerous:

- The model builder will be able to check by sight the logic of his model.
- The text can be given to other economists for advice.
- The full list of requested variables can be established, allowing to produce a complete transfer program.

Processing the equations through EViews will give interesting advice on several elements:

- The grammatical acceptability of equations will be checked-- for instance, the number of left and right parenthesizes—as well as whether each endogenous variable is computed only once.
- The variables-- the most important information will come from the list of exogenous variables. One might find elements which should have been determined by the model, according to its logic. In general, this will mean one has forgotten to state the associated equation. Also, some elements might appear which should not belong to the model. Normally, these are the products of typing errors.
- The block structure-- it decomposes the set of equations into a sequence of blocks, either recursive (each variable depends only on preceding elements) or simultaneous (some variables are used before they are computed). If one is going to succeed in estimating equations which follow the same logic as intended in the preliminary version, the block structure described at this stage will be already fully representative of the future one.
    - Abnormal simultaneities: a causal loop might appear, which is not supported by economic theory behind the model.
    - Abnormal recursive links: a block of equations containing a theoretical loop (the wage price loop, the Keynesian cross) can appear as recursive. This can come from a forgotten equation, a typing error, etc.

In any case, observing the causal structure of the model will give some preliminary information about its general logic, and its potential properties.

3.1.6.2 Consequences for Work Organization

In the general, the model builder will be confronted with a large set of series of more or less various origins. Optimal management strategy might appear to vary with each case, but in fact it is unique in its main feature: one must produce a file in the standard of the model building software and containing the series having a chance to be useful for the model.

Even if the global set of necessary series is produced and managed on the same computer or computer network, using the same software (the task of transfer will be simply made easier), it is essential that the model builder has control over the series he uses, and especially that he manages changes (in particular updates of series in current use). In interpreting a change in model properties (simulations, estimations), one must be able to dismiss a change in the data as a source, unless this change has been introduced knowingly by the model builder himself.[2]

Such an organization also makes the management of series easier. In particular, limiting the number of series in the bank, apart from the fact that it will save computer time and space, will make the set easier to handle intellectually.

Concerning the scope of the series, two extreme options can however be considered:

- Transferring in the model bank the whole set of series that have a chance (even if a small one) to become useful at one time to the development of the model.[3]
- Transferring the minimum, then adding to the set according to needs.

If a median solution can be considered, the choice leans strongly in favor of the first solution. It might be more expensive initially in both human time and the size of files, but it will prove generally a good investment as it avoids often a costly number of limited transfers and gives some stability to the bank as well as to its management procedures.

3.2 First Example: South Africa

The first task of the model builder is to define the logic of his model and its individual theoretical behaviors. This is done without accessing the data for the moment, even though of course one must have a rough idea of the elements available to avoid having to guess to too much at their value.

[2] This remark is a particular application of the general principle « let us avoid potential problems which can prove expensive in thinking time ».
[3] Even if they are not considered for actual model variables. For instance, one can be interested in comparing the capital output radio of the modelled country with those of other countries.

3.2.1 The Economic Formulations

For our example model, we shall limit our ambitions to a single market product. We shall also concentrate on the real sector, using a simple financial framework. But as we are considering an operational function, we shall describe the budget elements in as much detail as possible.

- Based on their production expectations and the productivity of factors, and possibly their relative cost, firms invest and hire workers to adapt their productive capacity. However, they exert some caution in this process, as they do not want to be stuck with unused elements.
- The levels reached in practice define potential production.
- Firms also build up inventories.
- Households obtain wages, based on total employment (including civil servants) but also a share of Gross Domestic Product. They consume a part of this revenue, influenced possibly by inflation, the risk of becoming unemployed, and the interest rate.
- Final demand is defined as the sum of its components: consumption, productive investment, housing investment, the change in inventories, and government demand.
- Imports are a share of local -"domestic"- demand .It is influenced by the competitiveness of imports compared to local products, and the supply available in the country.
- Exports are a share of world demand, depending on their competitiveness, and the available capacity, as the priority is satisfying local demand.
- Real supply is equal to real demand.
- Productive capital grows with investment, but is subject to depreciation.
- We shall introduce the following behaviors.
- Wages depend on local inflation, and possibly unemployment which affects the negotiating power of workers. The production price grows with costs, and optimizes profits according to a tradeoff between margins and quantities sold.
- The trade prices (imports and exports) depend on exporters' costs and the price of their competitors.
- The price of final demand balances the supply-demand equilibrium at current prices.
- Taxes are obtained by applying a rate to its base.

The above framework looks rather straightforward. We shall now address the above ideas in more detail.

3.2.1.1 The Productive Process

This part of the model-- one speaks often of "blocks"-- will not define production, but rather potential production (or productive capacity) as a function of available factors. Why not actual production itself? There are two ways to consider production:

- Actual local production, contributing with foreign exporters to the satisfaction of demand (both local and foreign) demand, in a share depending on relative prices and available capacities).
- Potential production, given by the production function, taking into account the level of factors (capital and labor), themselves chosen by firms according to their relative costs, expected demand, and profits conditions.

We want our model to follow the most logical causal sequence:

- Defining target capacity depending on profit conditions and expected demand.
- Choosing the optimal level of factors allowing this capacity.
- The actual levels will adapt, giving potential production.
- Global demand will follow, and will be shared between local and foreign producers to give actual production.
- Imperfect knowledge of future demand, technical difficulties, and concerns in a fast adaptation of factors will contribute to the creation of a gap between potential and actual value.

The comparison between actual and potential production will play an important role in some behaviors. This is the sequence that the model will describe, actual production being obtained late in the process, once demand is known (as in the small model).

This capacity for production will be measured:

- for employment, in man/years or man/quarters according to model periodicity
- for capital, at constant prices in the currency of the country.

The function can also include:

- energy consumption
- intermediate goods (like raw materials)

Capacities are generally defined in terms of value added, a more reliable notion as we have explained earlier. This means the two last elements are not taken into account, or rather their level will come automatically from value added itself.

The first issue concerns the logical link between capacity and factors. We have already seen complementary factors. For a given capacity, there is a single optimal process using a fixed combination of labor and capital.

Starting from an optimal combination, adding to the process a quantity of one factor does not increase capacity or allow using less of the other factor. This capacity is obviously optimal regardless of the relative costs. Labor productivity generally has some flexibility, and capital is the truly constraining factor as temporary and limited increases in labor productivity can be achieved (e.g. by increasing the number of hours worked).

This is the simplest option in its formulation, estimation, and understanding of properties. Operational models use generally more sophisticated frameworks:

- **Cobb-Douglas.** The elasticity of substitution is unitary; this means that if the ratio of the cost of labor to capital changes by 1%, the optimal ratio of capital to labor will change by 1% as well for a given capacity requirement.
- **CES (Constant elasticity of substitution).** The elasticity can take any fixed value (with the right sign). The CES option covers both others (with fixed elasticities of 0 and 1 respectively).

The framework also calls for a definition of the relative cost. The relative cost of labor and capital is not just measured by the ratio of the wage rate to the investment deflator. One also has to take into account the following:

- **Social contributions of firms.** They contribute to the cost of labor.
- **The interest rat**e. While capital is bought immediately[4], labor can be bought (rented) when the time comes. As such, a firm that has money can save it, and one which does not does not have to borrow.
- **The depreciation rate.** Capital wears out, while when a worker "wears out" through old age or sickness, he will leave and can be replaced by a new one at no cost except training (pensions have already been saved as a share of wages).
- **The future evolution of wages.** If wages are currently growing faster than inflation, firms can expect labor to become less competitive. The gain from having output transferred to fast developing countries becomes lower as they close the gap with developed ones. This applies in particular to present China.

One also has to take into account the possible changes in technology. The issue here is to decide if the technology decided at investment time (which defines the roles of labor and capital) can change later.

[4] Actually, some forms of capital (like buildings, computers or patents) can be rented or leased.

Basically, the options are:

- A single available technology (Clay-Clay).
- A technology chosen at installation time, with no later change (Putty-Clay). This means that the "complementary factors" option applies to factors once they are installed.
- A technology with a permanent possibility of change (Putty-Putty). The same substitution option applies to factors at any period.

We see no specific reason to modify the framework used by the small model for variations in inventories. More sophisticated formulations could use:

- A full error-correction framework, provided we knew the level of inventories.
- An influence of demand: if it goes up suddenly, some of it can be met by using inventories. This element will be difficult to introduce, as it calls for a negative influence, while value added has a positive one, and both elements are positively correlated. This means the over estimation of one coefficient can be compensated by over estimating the second too.
- An influence of prices: the more expensive the inventories, the shorter the time they will be stored.

We shall consider that the variations of employment do not transfer fully to unemployment. Job creation will attract to the labor market previously inactive persons, who shall take some of the jobs offered: the work force (employed + unemployed) will increase.

For instance, creating a firm in a low industrialized zone will allow housewives to combine employment with domestic work.[5] Or employees of a closing down factory will not necessarily remain in the labor market if their qualification is not required elsewhere.

The level of unemployment should also influence its dynamics. If it is high, the incentive to join the work force will be lower. Favorable employment prospects will lead young people living with their parents to start their working life. On the contrary, a depressed labor market will persuade aged workers to retire earlier (and they will be incited to). And some of the unemployed will stop looking for a job, and leave the work force.

Also, the higher the unemployment level, the higher the quality of the best unemployed. Observing the situation, the average unemployed people will lower their probability of getting a job, leading them to leave the work force.

On the contrary, at a low level of unemployment, the unemployed will feel that they stand a good chance over their competitors, most of them being either inefficient or not really looking for employment.

[5] Which is not considered as employment (maybe because it is not paid, and does not affect GDP, even if paid housework does).

This obviously corresponds to an error correction framework, leading to a target rate of unemployment (and also of participation of potential workers to the labor force, as we shall see).

3.2.1.2 The Price System

The role of prices in a model is essential but not so simple to introduce, even for a minimal model like the one presented above. In this case, several deflators have to be introduced simultaneously, associated with the elements in the supply-demand equilibrium:

- GDP
- Final demand
- Exports
- Imports

Plus:

- Wages (possibly including social security contributions)
- Deflators for each element in the decomposition of demand (consumption, investment, government demand)
- The price of foreign currency (the exchange rate)
- The prices of lending and borrowing (the interest rates)

Moreover, trade prices have to be defined including and excluding taxes. This distinction applies to external trade (for defining competitiveness and trade balance) and local demand (for defining final and intermediate consumption).

Not all these elements have to be estimated. Behaviors should be associated with:

- GDP (firms decide on the price at which they sell, once they take into account the cost of input).
- Exports (local exporters do the same)
- Imports (now we consider foreign exporters)[6]
- Wages (the result of a negotiation between workers and firm managers)

Final demand price should be used to balance supply and demand at current prices. The model gives a balanced set of four elements at constant prices, and three of the deflators have already been decided. The demand price should balance:

$$Pfd \cdot FD + Px \cdot X = Pq \cdot Q + Pm \cdot M$$

or

$$Pfd = (Pq \cdot Q + Pm \cdot M - Px \cdot X) / FD$$

[6] Remember we are building a single country model. The description of trade will be different with several connected countries.

Let us now address the links between prices. In the system, the deflators will depend on each other. For the time being, we will only give indications. A more detailed reasoning will come with actual estimations.

The GDP deflator depends on the wage rate, or rather the wage cost.

If wage costs go up, firms will have to increase prices to keep their margins. They do not have to do it immediately, and they are not obliged if they want to keep their competitiveness on the local and foreign markets (for exporting firms).

It actually might be better to use the global cost, including amortization of capital.

The wage rate depends on the consumption price, but maybe also on the value added price.

If prices go up, workers ask for a raise in wages to sustain their purchasing power. But again, firms are less liable to accept raises if they were not able to increase their own price.

Trade prices depend on the cost supported by the exporter, and on the price set by its competitors.

This means they have to maintain their margins and their competitiveness at the same time. This behavior is obviously based on production prices, the price at which they sell, which means the cost of intermediate consumptions has to be taken into account. For instance, a country having access to cheap oil will be able to export at lower prices, even at the same cost in value added (and the same margins). But this introduces a problem, as until now the single product feature allowed us to discard intermediate consumption, a variable difficult to manage as its value depends on the classification.

The behavior also has to apply to the same currency. If the export price uses the currency of the exporter, the price of its competitors measured in foreign currency has to be corrected by the exchange rate.

The price of demand depends on the price at which local producers and foreign exporters sell on the local market.

This uses the identity above.

Another important issue concerns the separation between the prices at which local firms sell on the local and foreign markets (the export price).

Two behaviors can be considered:

- The firms define both selling prices separately. Local firms start by defining a price for selling on the local market, using the above behavior. Then the export price will average this price and that of competitors.
- The firms define first a global selling price, allowing to reach a global margin rate, then they chose a combination of the two prices which meets this target. This means that a decrease in the export price (possibly designed to stay competitive facing foreign deflation) will have to be compensated by an increase in the local selling price.

The choice will have a strong impact on the price system. The second option will increase the intensity of the price-wage loop: if local costs go up, firms refuse to apply completely these costs to exports (as they do not want to lose their competitiveness), and maintaining global margins calls for a larger increase in local selling prices (which does not happen if targets are defined separately). This equilibrium is subject to external influences, either endogenous or exogenous.

- Endogenous
  - If labor productivity goes up, firms need fewer workers and can pay them more. They can also lower their prices.
  - If output is too low compared to capacities, firms can first lower prices to sell more (later they can adapt their capacities).
  - If unemployment goes down, workers can increase their demands without the risk of firm managers to look elsewhere.
- Exogenous.
  - VAT.
  - The other indirect tax rates, such as the tax on tobacco, gas, alcohol.
  - Tariffs.
  - The rate of social security contributions by firms.

If indirect[7] tax rates (e.g. VAT, tax on gas, cigarettes, and social contributions paid by firms) go up, then firms should adapt their price if they want to keep their margins.

It is quite important to separate these taxes in a model, for the usual reason: their base is different, and their impact on the economy also. VAT applies only to value added, but the most important feature is that it does not apply to exports (exporters can deduct it before they sell abroad), and they apply to imports. VAT on foreign cars is the same as on local ones, and applies to the total value. And when the car firm looks for electronic equipment, increasing VAT on this good will not change its decision on its origin as it can deduct VAT anyway. On the contrary, the other indirect taxes apply only to local productions, even though the impact of this difference is not so high, as imported goods are often taxed at the moment they are sold. For instance, the tax on tobacco applies also to imported cigarettes, and the tax on alcohol to imported whisky.

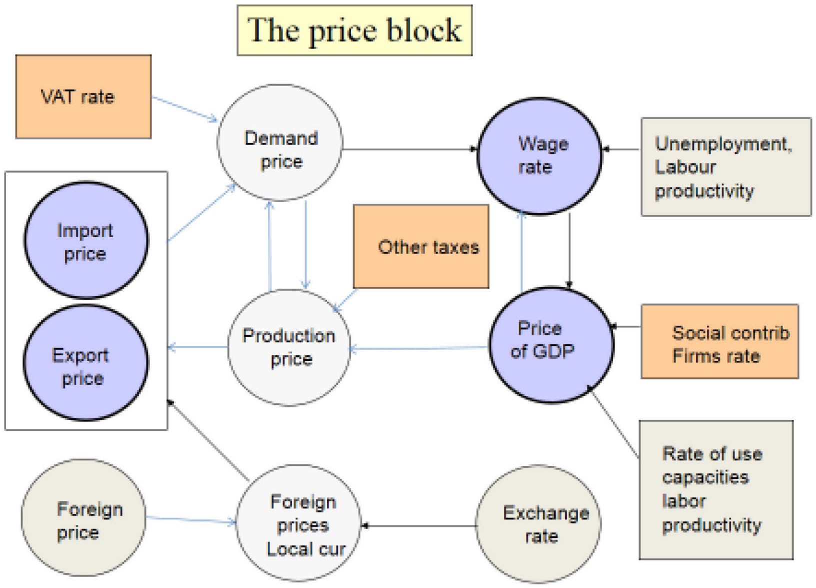
Concerning local tariffs, they are not deductible in the general case. This means that a change in their rate will affect directly the competitiveness of imported goods, unless the importer decides to compensate the effect by adapting its margins. As for tariffs applied to local products by foreign countries, they affect directly the competitiveness of exports. This means they have to be considered, even though their statistical value is not directly obtained from the national accounts.

[7] These taxes are called indirect because they are not paid directly by the ultimate payer, contrarily to income tax, corporate tax, etc.

Formalizing the role of taxes relies obviously on the rates, the variables decided by the state (or foreign states, for tariffs on local exports). These rates will affect deflators, and allow computing the amount of the tax once the base is known. It should be clear that the right way to formalize these taxes is to set the rate as an exogenous decision variable, and not to estimate the amount as some modelers might be tempted to do. This allows handling the decision easily, both in forecasts and shock analysis. And on the past, the technique is quite simple: the tax amount and the base are known, and this allows computing the rate, used as an exogenous ratio. The associated identity (tax = rate x base) will hold true. We shall see later how to handle these rates on the future. Obviously, the rate obtained will be different from the legal one—generally, lower. This technique is consistent with the general approach: make the government decisions exogenous, but identify first what represents the true decision.

- Prices can also influence real elements
- The selling price of local producers determines the quantities they will sell. This is also true of exporters, through the comparison between their export price and the price on the market on which they sell.
- The relative costs of labor and capital influence the choice of the factors in the productive process.
- More generally, ratios of prices affect the ratios of elements (or the shares in a total). For a given global consumption level, reducing the price of one good will increase its share.
- A higher inflation reduces the purchasing power of previous savings, calling for a larger reconstitution effort.
- And of course prices enter the definition of variables at current prices, when they are separated into volume and deflator (elements in the trade balance, wages...). For the elements in a sum, a different evolution of deflators will change the shares at current prices.

All this is described by the following graph.



3.2.1.3 The Account of Firms

We have already dealt with the supply side, defining the adaptation of production factors: employment and capital, to target capacity, as well as the decision on prices, based generally on the short term maximization of profits.

This means that most of the remaining equations will be definitions, describing the firms' account without calling for any theoretical elements. There are, however, two exceptions.

The first is the tax on profits, which should be again computed by applying a rate to a base. This is more complex than usual, however, as computing profits in a model is quite complex, and not all models are able to do it. Sometimes it is necessary to use a proxy, making the apparent rate more difficult to interpret. Additionally, the timetable for the tax calls for a dynamic equation, as the tax is not generally paid in the same period as the associated profits (but there can be a provision to pay immediately). So a formula describing the mechanism must be established. The tax on negative profits is not negative, but null, introducing a bias on the apparent rate.

The second is dividends paid by firms, which can be estimated or constructed through an identity (using a rate in the same manner as taxes). Again, one must decide on the dynamics, as dividends follow the profits. Also, the beneficiary of dividends has to be identified (sharing must be done between the five usual agents).

Of course, the complexity of formulations and even the identification of elements such as dividends depends on the role of the model, whether it is used by researchers trying to answer global theoretical issues or by policy advisers addressing in detail the evolution of the next state budget.

3.2.1.4 The Behavior of Households

Households obtain revenue from several sources, the main ones being:

- wages
- revenue of individual workers
- social benefits of various kinds
- interest from loans
- dividends
- renting lodgings to other households (a service)[8]

They use this revenue in turn to:

- pay income tax
- consume various goods and services
- save-- in particular in housing, but also in deposits, bonds, stocks and goods (e.g. art)

To be considered operational, even a single product model must use some detail, as the economic processes through which these revenues are obtained and the consequences of spending decisions are quite different from each other.

Another principle of modelling: *favor the detail which allows separating behaviors.* This has several implications for revenue and expenditures. Regarding revenue:

- Wages paid by firms should be the product of an average wage rate (coming from the price block) by the number of workers (from the production block).
- The number of civil servants will generally be exogenous, but not the wage rate, which can be different from firms'.
- Wages paid by households (mainly for housekeeping) can be identified or not, according to the type of model.
- Social benefits are generally separated in five types: sickness, family subsidies, unemployment benefits, invalidity from working accidents, pensions.

It is clear that each of these elements depends on inflation, but at different degrees. Most of them depend on population, and often a given type of population. For instance, the number of children, the number of people having reached retirement age, or of unemployed. All of them depend on economic activity, again in a variable way. For instance, unemployment benefits decrease with GDP, working accidents increase,

[8] It is strange to consider that if a household buys the apartment it is renting, the service disappears and GDP decreases. For that reason, housing owners are considered by national accounts as paying themselves a fictitious rent.

and pensions should increase (in principle) with the revenue from the contributions which finance them. They also depend on a decision made by the state-- the purchasing power is maintained). This means that an operational model should try to separate these items, in order to take into account their differences in behavior. In this way, the model will show the change in benefits with the number of beneficiaries and the change in benefits with the decision.

Interest will be described globally in a subsequent paragraph. Let us only stress for now that, for households, the interest rates (lending and borrowing) can be deviate from market values through state intervention. In France, for example, a limited amount of savings benefits from a higher guaranteed rate, and borrowing to buy housing can be done at a lower rate (0% in some cases).

As to the revenue from housing (rent), its role in a model is limited as it mostly represents a transfer from households to other households. For owners of property, it is even a transfer within the same household. There are reasons to consider it, however-- it can be subject to taxation, and it enters GDP.

One should not consider marginal elements, such as lottery winnings, inheritance, donations, and fines.

Finally, one can formalize the transfers from abroad or to abroad. For developing countries, remittances can represent a sizable share of household revenue (e.g. more than one third of GDP for Tajikistan). For a single country model they should be exogenous, perhaps even in current terms (a notable exception to the general principle).

Moving on to expenditure, the income tax should be computed as a rate applied to revenue before tax, obtaining the historical values of the apparent rate by dividing the amount by the base. The model will then get the tax by applying the exogenous rate to the base. The base poses the usual dynamic problem: the tax can be paid after the revenue is obtained, with a provision mechanism.

Applying an average rate to all households can be acceptable for forecasts, which allow this rate to change with time, but less so for the shocks addressed to a category of households at one extremity of the spectrum: in a traditional macroeconomic model, a decrease in the tax on large or an increase in benefits for the poor, of the same ex ante size, will have the same ex post consequences.[9] To eliminate this error, an ad hoc correction has to be made on the savings rate itself. This problem appears in most models, coming from the fact is that the tools to solve it are not available. National accounts separate firms using the goods they produce, but do not distinguish between households for variables such as the level of revenue. Some surveys address the problem, and they could be used to create specific data. This means some solution might be found, but without doubt at a high cost. Actually, the same problem arises if one wants to separate firms not according to sectors but size, considering that small firms act differently from large ones.

[9] Of course, the impact on consumption will be higher if the increase concerns the poor.

Once the disposable income is known, all that remains is to separate it into consumption and savings, considered as whole in most models (for multi-product models the situation will be more complex). The most common technique is to compute consumption first, as a ratio to revenue, then savings as a residual. We shall develop this with estimations.

Consumption is generally determined at constant prices (which means in purchasing power). The usual determinants are as follows:

- The level of revenue (measured also in purchasing power). The higher the revenue, the higher the consumption level-- but the lower the share of consumption. (The poor do not save, and remember that buying a house is considered as savings.)
- The recent evolution of revenue. Households take some time in adapting their behavior to an increase (or decrease) in revenue. And a sudden hike (especially if it is destined to be permanent, like a promotion) can lead them to invest in housing, which can actually decrease consumption for a while.
- Inflation (the "real holdings" effect). Present savings contain a large share of monetary elements (deposits, bonds with fixed rates, etc.). Current inflation reduces their purchasing power, which has to be complemented by additional savings. The effort is proportional to the inflation level.
- The unemployment rate. For employed workers, an increase in the chance of losing their jobs (measured more by the change in the rate than its value[10]) leads them to save a larger share of their present revenue, if they want to optimize their utility across time.
- The (short term) interest rate. In general, people prefer satisfying a given need now than later. But this has a cost, the interest they have to pay. The lower the rate, the more they will indulge in immediate consumption.

This is particularly true for durable goods: if a household wants to watch flat screen TV (and thinks that after its purchase, in its whole life it will have enough resources to afford a set) the only reason for not buying one right now and increasing its satisfaction permanently is the actualized cost, which is lowered with a decrease in interest rates. What the household has to consider is not the cost of the good, but the cost of making its acquisition earlier. If the good is perfectly durable, and can be sold back at its original value at constant prices, it is comparable renting the good. If the interest rate is divided by two, the "price" of the good is divided by two.

[10] Although the actual rate plays also a role: a higher value implies a higher turnover, and a high risk of participating in the turnover.

For non-durable goods, the situation is different. The household has already optimized its consumption over time. If the interest rate changes, it might be tempted to consume earlier, but if the marginal utility of the good is decreasing fast, the pattern of consumption will not be much affected. A person dreaming of visiting the pyramids, and saving for that purpose, might make the trip earlier but will not do it again.

What matters is the real rate:

- They allow comparing goods at constant prices.
- If households assume their revenue will grow with inflation, they will optimize in real terms.

Once consumption is determined, savings are computed as a residual, and generally as a global element. This option can be discussed, as different kinds of savings can be assumed to follow different behaviors.

In particular, housing investment is negatively affected by interest rates-- a specific rate, but one can assume it follows the global rate-- while financial savings are positively affected. Buying a house calls for obtaining a given good and asking another agent to provide the collateral in return for interest. Buying a bond means lending collateral to another agent to use it as a spending tool (maybe to buy a durable good) in return for interest, but this time in the other direction.

3.2.1.5 External Trade

In a single country model, the rest of the world is exogenous. This means that we consider only influences from the world to the country, and not the other way around.

Of course, this is not really exact even for the smallest of countries (or in that regard for a region, a town, or an individual): by increasing your consumption and so local production, you create a fraction of a job, a small amount of household revenue, and again more consumption.

What we consider is that the influence is too small to have a sizable effect, and that the cost of producing and running a model describing it is too high compared to the gain in the accuracy of results. This is essentially true for smaller or medium-sized countries like Latvia or Bolivia, less so for larger countries like France, and quite untrue for the USA or the European Union considered as a whole. For instance, when we use the MacSim world model for a shock analysis, the French Keynesian multiplier for 2000 was 1.3 if we run the full model, but only 1.1 if we run the French model by itself. The iterative feedbacks of German imports from France, coming from the increase of German exports, will have the largest share in the difference. Considering the evolution of world trade, the present difference should be even wider.

This means that the exchanges of the country have to be considered from the point of view the country:

- Exports are the share of production of goods and services which is sold by the country to the rest of the world.

- Imports are the share of local demand for goods and services which is not produced in the country, but bought from the rest of the world.

Both elements will be computed using the currency of the country, but using constant prices will mean using the exchange rate of the base year; so the currency issue is not relevant, introducing only a scaling by a constant factor.[11]

The trade elements having the same nature, however, their logical determinants will be the same. The main difference will come only from the relative size of the two markets (buyer and seller) in the trading operation: the single country's importance (or GDP) will always be much lower than that of the rest of the world, although this is less obvious again if we model the USA or the European Union as a whole.

These elements will be:

Demand. For a country to sell a given good to a partner country, demand for this good must be present, part of this demand must be addressed to the world market, and the quality of local products must appeal to the importing country. For instance, French exports of wine will depend on the world demand for wine, and the natural preference of importing countries for foreign wine (starting with their status as wine producers) and French wine in particular.

Defining demand introduces two problems. For imports, we have already seen that including intermediate consumption in the supply-demand equilibrium (thus considering production on one side and total local demand on the other) is quite a problem for models, as the level of intermediate consumption depends on the number of steps in the production process. The single product feature has until now eliminated the need for considering intermediate consumption. Imports, however, contain intermediate goods, whether they represent energy (oil, gas, even electricity) or primary goods (from untreated wood to electronic components), and these intermediate goods are necessary to exports.

A simple solution is to consider the ratio of intermediary consumption to value added. Looking at the figures, we can indeed observe that the technical coefficients (the number of units needed to produce a unit of value added or GDP) is rather constant. We just have to consider a composite demand as the sum of final demand itself, and intermediate consumption as a function of GDP (or rather value added, as intermediate consumption excludes VAT).

In countries in which the trade balance is more or less in equilibrium, we might consider using a combination of final local demand and exports.

Price competitiveness. To decide whether to buy a good from a local or foreign producer, a country will compare the local price with the foreign exporters' price. To choose among potential sellers, the importing

[11] This is only true if we consider a single rest of the world, or we measure it in a single currency. More on this later.

country will consider their relative price at a given quality (remember that the deflators consider goods at the same quality level, an increase in quality improving the value at constant prices).

We generally observe that the relative price is less of an issue when the buyer contemplates buying local or foreign goods than when he has to choose between foreign sellers. This follows economic logic: local goods are supposedly designed for the local market, and some goods are not or hardly substitutable (e.g. local bus tickets or newspapers).

This means in our case that the sensitivity of exports to price competitiveness should be higher than that for imports. Exports depend on world demand in the world market, and once a country has decided to import, the price will play a more important role than in the import decision itself.

Of course, measuring competitiveness must use deflators defined in the same currency. It can be any currency, as applying the same exchange rate to both elements of the ratio will not change its value. In the case of exports, this means that measuring their deflator in local currency calls for a foreign price measured in the same units. As the exchange rate is identified, this foreign price will be endogenous as the product of two assumptions: the foreign price in foreign currency, and the correcting exchange rate (a deflator). It is perhaps more logical, and equivalent in practice, to consider both prices in foreign currency, the local one being corrected by the symmetric exchange rate.

The available capacities. The third element is the potential to supply additional demand, which means the presence of available productive capacities. The relevant variable is naturally the rate of use of capacities, independent from the size of the economy.

The choice of this option is not so straightforward, however. One could argue that as long as the rate is lower than one, additional demand can be satisfied. We have already shown that this is not true: demand concerns a wide range of products, and one cannot generally be substituted for another, in particular in the short term. Some products may see their capacity completely saturated.

Let us explain the process again, this time in an import-oriented way.

The average rate of use is based on a distribution of rate values, from zero (hopefully a few cases) to one (probably a sizable number). When global demand increases, it addresses a range of firms and in variable intensity. Some of these demands will be addressed to firms already unable to provide more, and some others will make them reach that level. The proportion of firms working at full capacity will grow. Of course, in some cases, another available product can represent a substitute; but the most simple option is to import the same product, as the missing product should be available somewhere in the world (maybe at a higher price, but this should be treated by the price competiveness).

The “missing” demand increases with the share of firms which cannot increase their production, having reached capacity.

Of course, this phenomenon applies essentially in the short term, as firms will react by investing, which will increase capacity and close the output gap with time. But this process can be slow, even if full adaptation should be obtained in the very long run.

But if we follow the above reasoning, we observe the following:

- The larger the country, the lower the probability that a given absolute but also *relative* increase in demand will face local supply problems. This increase in demand will be more diversified, and the available capacities will be more diversified too.[12]
- In our case, the rest of the world should not face any supply problem, which means that for both our country's imports and exports, only the local rate of use should be taken into account.

A last condition can appear for the exporting country. If the world requires a given good, the characteristics of that good produced in the country must also be adapted to the particular demand, which can be quite specific. For instance, facing an increase in the foreign demand for cars, a country might be able to supply them at a competitive price, but the type of cars they propose might be unsuitable. This might concern the size, the sophistication, the powering energy, the security features, and so on. One cannot consider going in such a detail in a model, even if the data was available (which it is not).

Unfortunately, finding an element describing this feature is less straightforward than above, especially for a single product model. The simplest idea is to use the age of capital, assuming that a younger productive process will be better adapted to present demand.[13] For instance, a recently built car factory might follow market trends in producing smaller cars, or more energy efficient ones. The age of capital can be derived simply from the chronology of investment and depreciation, if we consider that this depreciation applies equally to all generations of present capital, or that capital disappears brutally a given number of years after its implementation. Another assumption leads to more complex but manageable formulas.

3.2.1.6 The Budget

Fully and consistently describing the government budget is an absolute requirement in operational models. This is true even if the model is not going to be used by government advisers but by experts in applied economics. The general goal of these researchers is to assess the consequences for the economy from government decisions, external events, or structural changes, considering the most probable impact or the range of possibilities and possibly under different model formulations (like different options on the interest rate). The approach might be more or less applied-- the advisers might try to produce an image of the next

[12] This would not happen if the additional demand was in a specific good.
[13] Especially foreign demand as its role is increasing with time, so its influence on the nature of investment will be higher in later periods.

budget, to be presented to the Parliament, and the scientists will try to see how the adoption by the Central Bank of a Taylor rule will stabilize the economy-- but the tool required is quite similar.

As we have stated above, the best way of defining the associated equations is to build identities, computing an endogenous revenue or expenditure as the product of an endogenous base by an exogenous rate. The equations will hold true over the past, and the modeler will be responsible for (and allowed to) establish future assumptions on the rate. He does not have to keep this rate constant and can rely on an estimated trend as a starting base, but the final decision will be his.

This technique answers to the following objection: if we consider VAT, even with constant legal rates, the apparent rate will change (grow) with the affluence of households, able to increase the share of highly taxed products in their consumption. One solution is to establish a trend, used as a base value, and to deviate from this trend as a policy decision.

If these principles are followed, it will be possible to produce a table showing the evolution of all budget elements in current terms and in GDP points, both features required for presentations.

Another important principle of modelling: if you cannot chose between the possible presentations for a given concept (value at constant prices, at current prices, growth rate, and ratio to another variable), just look at how this concept is presented in economic publications written for the general public). Alternatively, wait until you will have to use the figures in your own presentations, then measure your reaction and that of the public.

3.2.1.7 Financial and Monetary Elements

In any model, this represents the most variable and controversial part. The first models had little or no financial equations. Even at this stage, the financial block can be limited to the definition of a few rates, and their impact on the real sector (these rates can even be exogenous, generally in real terms). On the contrary, this block can be so developed that the purpose of defining a real sector can be considered as a way to complete the links between financial elements, for instance describing the creation and origin of additional lending if a decrease in interest rates draws investment upward.

In our opinion, even a real side oriented model should include:

- A base interest rate set by the Central Bank of the country.
- A short and a long term rates in the currency of the country.
- An average rate on current net borrowings.
- A rate on the present debt, being computed from the chronology of past rates, perhaps as an autoregressive function.

- One or several foreign rates, applied to borrowings in foreign currency both in the country and in the rest of the world.
- The net interests paid by all (five) agents, considering two currencies for the interests paid to the Rest of the World.

An example of this framework will be presented soon.

From this basic option, developments can consider:

- Identifying the debt of agents (or their financial holdings).
- Separating it into currencies (local, US Dollars, maybe Euros for non EMU countries).
- Separating it into short term and long term.

In addition, one or more forms of money supply can be formalized.

Most of these equations should be established as identities, based on available data or assumptions. Exceptions can concern:

- The Central Bank rate, following perhaps a Taylor rule, but not necessarily. Actually, the same model should allow several options (using a separating parameter).
- The short term and long term rates could include a risk premium, depending for instance on the current budget deficit or its most recent values.
- The spread between long and short term could depend on growth expectations (more true if they are partly or totally rational) and the health of the local economy.

We shall stop here, as financial issues are less a purpose of this publication.

3.2.2 Defining the Model Equations

We shall now define a full set of equations which makes explicit the framework we have just defined. The endogenous variables will use uppercase characters, while the exogenous will use lowercase.

3.2.2.1 The Production Block

GDP at constant prices balances the supply-demand equilibrium

[1] GDPM = FD + X - M

Value added excludes value added tax and tariffs

*[2] Q = GDPM - r_vat0 * FD / (1 + r_vat0)*

Capacity depends on employment and capital

[3] LOG(CAP) = f(LE, K)

The rate of use shows how much capacity is actually used for production

[4] UR = Q / CAP

Employment depends on value added and the « normal » productivity of labor, with a possible inertia.

[5] LF = f(Q, lpt, LF(-1))

Wage earners are a share of firms' employment

*[6] LW = r_lw * LF*

Total employment includes civil servants

[7] LT = LF + lg

Labor productivity

[8] LP = Q / LF

Productive investment depends on value added, the rate of use (output gap), previous values, and possibly the profits rate and the interest rate.

[9] IP = f(IP(-1), K, K(-1), Q, UR RPROF)

Capital is the sum of the remaining share of remaining previous capital and investment

*[10] K = K(-1) * (1 –rdep) + IP*

The change in inventories depends of value added.

[11] IC = f(Q, lagged values)

The work force depends on employment and population in age of working

[12] POPAC = f(LT, pop65, lagged values of POPAC, LT and pop65)

Unemployment

[13] UN = POPAC - LT

Its rate

*[14] UNR = UN / POPAC * 100*

3.2.2.2 The Prices

The wage cost per unit produced includes social security contributions by firms

*[15] COSTW = WR * (1 + r_scf) / LP*

The deflator of value added depends on the wage cost, and the rate of use (output gap).

[16] PQ = f(WCOST, UR)

The production price weights the prices of value added and the demand price excluding tax (a proxy for intermediate consumption).

*[17] PP = (PQ + tc * PFDXT) / (1 + tc)*

Final demand at current prices balances demand and supply.

[18] FDV = GDPMv + Mv - Xv

The final demand deflator.

[19] PFD = FDV / FD

The final demand deflator ecluding taxes

*[20] PFDXT = PFD * (1 + r_vat0) / (1 + r_vat)*

The detailed demand deflators use ratios.

*[21] PCOH = r_pcoh * PFD*

*[22] PIP = r_pip * PFD*

*[23] PIG = r_pig * PFD*

The wage rate depends on CPI and the value added deflator, labour productivity and the unemployment rate.

[24] WR = f(PCOH, PQ, LP, UNR)

The unitary cost includes a given share of capital.

*[25] COST = (WR * LF *(1 +r_scf) +c_cost * PIP *K(-1)) / Q*

The export price depends on the production price, the foreign price in local currency, and a time trend.

*[26] PX = f(PP, ppx * ER, t)*

The import price too, with reversed roles

*[27] PM = f(ppx * ER, PP ,t)*

The exchange rate is exogenous in current or constant terms.

[28] ER = erx

Or

[28] ER = f(PCOH, ppx)

The short term interest rate can be exogenous in real or constant terms, or follow a Taylor rule.

[29] IRS = irsx

Or

[29] IRS = f(irsr, PCOH)

Or

[29] IRS = f(irst, PCOH, UR)

The long term interest rate is a moving average of the short term one.

[30] IRL = f(IRS)

The average rate weights both rates.

[31] IR = f(IRS, IRL)

The rate on the past debt is corrected by new borrowings.

[32] IRM = f(IRM(-1) IR)

The relative cost follows the traditional formula.

*[33] RELC = WR * (1 + r_scf) / PIP / IR / 100 - @pchy(PCOH) + rdep)*

where @pchy represents the yearly growth rate in EViews.

3.2.2.3 Households

Household revenue

The exchange rate is applied to remittances.

*[34] REM = remx * ER*

A part of household revenue is exogenous in constant terms. .

*[35] REVX = r_revx * PFD*

Another part is proportional to value added.

*[36] REVQ = R_REVQ * QV*

Social benefits per head are exogenous in purchasing power.

*[37] SOCB = socbr * PCOH * popt*

Civil servents obtain the average rate.

*[38] WAGEG = WR * lg*

Total wages.

$$[39]\ WAGE = WAGEF + WAGEG$$

Workers pay a share of their wages as contributions

$$[40]\ SCW = r\_scw * WAGE$$

Total household income

$$[41]\ HI = WAGE - SCW + REVQ + REVX + SOCB + REM$$

The income tax is proportional to income.

$$[42]\ ICT = r\_ict * HI$$

Disposable nominal and real incomes.

$$[43]\ HDI = HI - ICT$$

$$[44]\ HRDI = HDI / PCOH$$

Housing investment is a share of disposable income.

$$[45]\ IH = R\_IH * HRDI$$

Household consumption depends on real disposable income, unemployment, inflation, the real short term interest rate, and past values.

$$[46]\ COH = f(HRDI, UNR, PCOH, IRS, t)$$

3.2.2.4 The Account of Firms

Value added at current prices.

$$[47]\ QV = PQ * Q$$

GDP at current (market) prices.

$$[48]\ GDPMV = QV + VAT + TARIFF$$

The GDP deflator

$$[49]\ PGDPM = GDPMV / GDPM$$

The wages paid by firms

$$[50]\ WAGEF = WR * lW$$

Subsidies to firms are proportional to value added

$$[51]\ SUBS = r\_subs * QV$$

The margins of firms.

$$[52]\ MARG = PQ * Q * (1 + r\_subs - r\_oit) - WAGEF * (1 + r\_scf)$$

The margins rate.

$$[53]\ RMARG = MARG / Qv$$

The tax on profits

$$[54]\ IFP = (MARG - REVQ) * r\_ifp$$

The interests depend on their present value, the interest rates and the present deficit.

$$[55]\ NIF = f(NIF(-1)\ ,\ IRM,\ IR,\ FCAPF)$$

For profits, we substract the share of households, the tax on profits and the interests paid.

$$[56]\ PROF = MARG - REVQ - IFP - NIF$$

The profits rate (two definitions).

$$[57]\ RPROF = PROF / (PIP * K(-1))$$

$$[58]\ RPROB = MARG / (PIP * K(-1))$$

The financing capacity.

$$[59]\ FCAPF = PROF - PIP * IP - PFD * IC$$

Total investment

$$[60]\ I = IP + IH + ig$$

3.2.2.5 External Trade

The import price including tariffs

$$[61]\ PMT = PM * (1 + r\_tar) / (1 + r\_tar0)$$

where r_tar0 is the base year rate (as PMT and PM are deflators, the correction represents the change from the base year).

The import price competitiveness.

$$[62]\ COMPM = PMT / PP$$

Final demand.

$$[63]\ D = COH + IP + IH + IC + cog + ig$$

The intermediate consumption.

$$[64]\ CI = TC * Q$$

Total local demand.

$$[65]\ TD = FD + CI$$

Imports depend on total demand, the rate of use, and price competitiveness.

$$[66]\ M=f(TD, UR, COMPM)$$

Export price competitiveness compares the export price (including tariffs) to the foreign price in the same currency.

$$[67]\ COMPX = PX * (1 + r\_tarx) / (1 + r\_tarx0) / (PPX * ER)$$

Exports follow world demand, corrected by price competitiveness and the rate of use.

Imports at current prices.

$$[68]\ MV = PM * M$$

Exports at current prices.

$$[69]\ XV = PX * X$$

The ratio at current prices.

$$[70]\ RCVAL = XV / MV$$

The ratio at constant prices.

$$[71]\ RCVOL = X / M$$

The terms of trade.

$$[72]\ TTRAD = PX / PM$$

The trade balance.

$$[73]\ TRB = XV - MV$$

The interests paid to the rest of the world in local currency depend on past values, the interest rate on past debts, the new interest rate, and the trade balance.

$$[74]\ NIXD = f(NIXD(-1), IRM, IR, TRB)$$

In foreign currency, it includes the exchange rate and the foreign interest rate.

$$[75]\ NIXX =f(NIXX(-1), IRMX, ER, TRB)$$

Total interests paid.

$[76]\ NIX = NIXD + NIXX$

Financing capacity of the country.

$[77]\ FCAPX = TRB - NIX$

3.2.2.6 The Government Account

Value added tax.

$[78]\ VAT = r\_vat * PFD * FD / (1 + r\_vat)$

Social security payments by firms.

$[79]\ SCF = r\_scf * Wagef$

Other indirect taxes.

$[80]\ OIT = r\_oit * (Qv - oit)$

Tariffs.

$[81]\ TAR = r\_tar * Mv$

Social security payments by Government.

$[82]\ SCG = r\_Scg * WAGEG$

Government revenue.

$[83]\ REVG = SCF + ScCG + SCW + OIT + IFP + ICT + VAT + TAR + r\_revg * QV$

Government investment at current prices.

$[84]\ IGV = ig * PIG$

Government consumption deflator.

$[85]\ PCOG = PFD * r\_pcog$

Government consumption at current prices.

$[86]\ COGV = cog * PCOG$

Total Government demand.

$[87]\ FDGV = COGV + IGV$

The interests depend on their present value, the interest rates and the present deficit.

$[88]\ NIF = f(NIF(-1)\ , IRM, IR, FCAPF)$

The Government expenses.

$$[89]\ EXPG = FDGV + WAGEG + SUBS + SOCB + NIG + SCG + r\_expg * QV$$

The financing capacity.

$$[90]\ FCAPG = REVG - EXPG$$

The financing capacity in GDP points

$$[91]\ FCAPGP = 100 * FCAPG / GDPMV$$

3.3 The Second Task: Obtaining the Data

3.3.1 The Data Needed by the Model

Considering the previous elements, we can define the set of data we need (both endogenous and exogenous series).

Name	**Equation and definition**
CAP	Eq03 Productive capacity
CI	Eq64 Intermediate consumption
COG	Exog Government consumption (real)
COGV	Eq86 Government consumption (current)
COH	Eq46 Household consumption
COMPM	Eq62 Imports competitiveness
COMPX	Eq67 Exports competitiveness
COST	Eq25 Unitary cost of wage and capital
COSTW	Eq15 Unitary wage cost
ER	Eq28 Exchange rate
ERX	Exog Exchange rate (exogenous)
EXPG	Eq89 Government expenditures
FCAPF	Eq59 Firms financing capacity
FCAPG	Eq90 Government Financing capacity
FCAPGP	Eq91 Government Financing capacity (GDP points)
FCAPX	Eq77 Rest of the world Financing capacity
FD	Eq63 Final domestic demand (real)
FDGV	Eq87 Government demand
FDV	Eq18 Final domestic demand (current)
FDXR	Exog Residual demand
GDPM	Eq01 Gross Domestic Product Market
GDPMV	Eq48 Gross Domestic Product Market
HDI	Eq43 Household disposble income
HI	Eq41 Household Income
HRDI	Eq44 Household disposable income

I	Eq60	Investment
IC	Eq11	Intermediate consumption
ICT	Eq42	Income tax
IFP	Eq54	Tax on firms profits
IG	Exog	Government investment
IGV	Eq84	Government investment
IH	Eq45	Housing investment by households
IP	Eq09	Productive investment
IR	Eq31	Interest rate, average on new borrowing
IRL	Eq30	Interest rate, long run
IRM	Eq32	interest rate, average on current debt
IRMX	Exog	interest rate, average on current debt, exogenous
IRS	Eq29	Interest rate, short term
IRSR	Exog	Interest rate, short term, real
IRST	Exog	Interest rate, Taylor residual
IRSX	Exog	Interest rate, short term exogenous
IRX	Exog	Interest rate, foreign
K	Eq10	Productive capital
LF	Eq05	Trend of labor productivity
LF_EC	Exog	
LG	Exog	Employment of Government
LP	Eq08	Productivity of labor
LT	Eq07	Employment, total
LW	Eq06	Wage earners
M	Eq66	Imports (real)
MARG	Eq52	Firms margins
MV	Eq68	Imports (current)
NIF	Eq55	Interests paid by firms, net
NIG	Eq88	Interests paid by Government
NIG_ER	Exog	Interests paid by Government
NIX	Eq76	Interests paid by the rest of the world
NIXD	Eq74	Interests paid by the rest of the world, local currency
NIXX	Eq75	Interests paid by the rest of the world, foreign currency
OIT	Eq80	Other indirect taxes
PCOG	Eq85	Deflator of Government consumption
PCOH	Eq21	Deflator of household consumption
PFD	Eq19	Deflator of final demand
PFDXT	Eq20	Deflator of Final demand, excluding VAT
PGDPM	Eq49	Deflator of Gross Domestic Product
PIG	Eq23	Deflator of Government investment
PIP	Eq22	Deflator of Firms investment
PM	Eq27	Deflator of imports
PMT	Eq61	Deflator of imports including tariffs

POP1564	Exog	Population in age of working
POPAC	Eq12	Work force
POPT	Exog	Population, total
PP	Eq17	Deflator of Production
PPX	Exog	Deflator of foreign production
PQ	Eq16	Deflator of Value added
PROF	Eq56	Firms profits
PX	Eq26	Deflator, exports
Q	Eq02	Value added (real)
QV	Eq47	Value added (current)
R_EXPG	Exog	Residual on Government expenditures
R_ICT	Exog	Income tax rate
R_IFP	Exog	Rate of the tax on Firms profits
R_IH	Exog	Ratio of housing investment to revenue
R_LW	Exog	Share of wage earners in firms employment
R_OIT	Exog	Other indirect taxes rate
R_PCOG	Exog	Ratio of the Government consumption price to the global demand deflator
R_PCOH	Exog	Ratio of the household consumption price to the global demand deflator
R_PIG	Exog	Ratio of the Government investment price to the global demand deflator
R_PIP	Exog	Ratio of the firms investment price to the global demand deflator
R_REVG	Exog	Residual on Government revenue
R_REVQ	Exog	Other household revenue based on GDP
R_REVX	Exog	Other household revenue not based on GDP
R_SCF	Exog	Rate of social security contributions paid by firms
R_SCG	Exog	Rate of social security contributions paid by Government
R_SCW	Exog	Rate of social security contributions paid by households
R_SUBS	Exog	Rate of firms subsidies to Value added
R_TAR	Exog	Rate of local tariffs
R_TARX	Exog	Rate of foreign tariffs
R_VAT	Exog	VAT rate
RCVAL	Eq70	Ratio of exports to import (current)
RCVOL	Eq71	Ratio of exports to import (real)
RDEP	Exog	Depreciation rate of capital
RELC	Eq33	Relative cost of labor and capital
REM	Eq34	Remittances
REMX	Exog	Remittances
REVG	Eq83	Government revenue
REVX	Eq35	Household revenue, non GDP based
RMARG	Eq53	Margins ratio of firms
RPROB	Eq58	Profits ratio of firms, second definition
RPROF	Eq57	Profits ratio of firms
SCF	Eq79	Social security payments of firms
SCG	Eq82	Social security payments of Government

SCW	Eq40	Social security payments of workers
SOCB	Eq37	Social benefits
SOCBR	Exog	Social benefits
SUBS	Eq51	Firms subsidies
T	Exog	Time
TAR	Eq81	Tariffs
TC	Exog	Technical coefficient
TD	Eq65	Total local demand
TRB	Eq73	Trade balance
TTRAD	Eq72	Terms of trade
UN	Eq13	Unemployment
UNR	Eq14	Unemployment rate
UR	Eq04	Capacity utilization rate
URD	Exog	Target capacity utilization rate
VAT	Eq78	Value added tax
WAGE	Eq39	Wages
WAGEF	Eq50	Wages paid by firms
WAGEG	Eq38	Wages of civil servants
XV	Eq69	Exports (current)
REVQ	Eq36	Household revenue, GDP based
WR	Eq24	Wage rate

However, some of the series can be computed from other series (e.g. the deflators are obtained by dividing the series at current and constant prices). The series really needed are highlighted in blue.

We will now look for this data. Considering the model specifications, we are not prepared to reduce its size, and if some data is not available we are ready to imagine an assumption (reasonable) to determine them.

In our case, the main potential sources are:

- A local statistical institute.
- An international organization.

The first task of the modeler is to identify all of these sources, by himself or helped by some other individuals or organizations in pursuit of the same goal. It can always be helpful to reach out to several sources for the same variable, if only to check the reliability of the data, or the fact that it has been identified correctly (for instance one can observe that savings did not include housing investment, or that a deflator did not include taxes).

But in all cases one should identify a major source, producing consistent data and used as the backbone for the data set. The additional series should be used only to replace missing information. For instance if the detail is missing for demand deflators, one should use the global value from the central data set, and use a rule of three to integrate external, more detailed information.

This is true in particular if available sets have a different base year. The model should have a single one (even if most variables are used by their logarithms). It is very inconvenient (and a source of error) to have to adjust some of the series.

3.3.2 The South African Case

During our initial research we have not been able to access the South African Statistical Institute. We are managed to do it since (http://beta2.statssa.gov.za/) but the format and scope of the data we obtained is clearly not suitable to the building of our model. Instead we have relied on international organizations, first of all the World Bank, which provides a very large data set, comprising 1300 series per country, and 152 countries or groups. Of course, some of these series, even if present in the file, do not have a single valid value, but nevertheless this is an extremely interesting source.

Of course, we will not need the 1300 series for our model, only a small subset. However, we shall see that this huge set is not sufficient to provide us with all the information we need.

We will not give the definitions of all the series available (they are given as an annex) and only refer to the ones used, at the time they are used.

In this part of the document, we will not state all the computations we have performed (this is left to the user's manual). We will just state the problems we have met, and we way we have treated them.

3.3.2.1 The Lacking Series

Let us start with the missing series. Although the World Bank provides a lot of information, sometimes in great detail, some very important elements are not described in the data set. They are, in decreasing order of importance:

- **Employment and wages.** These series are clearly required, as they enter the wage-price loop, the production function and the households and firms accounts.
- **Capital.** This is required too, but not readily available in most data banks. As we shall see, there are ways to compute it, depending on the related information available.
- **Intermediate consumption.** This is needed to compute total demand (which defines imports) and the production price (which defines the trade prices and competitiveness).
- **Housing investment.** This is a part of demand.
- **Non-wage revenue of households**. This enters household revenue and influences consumption.
- **Social contributions.** This affects the revenue of all agents, and the cost of labor (thus the value added deflator and the capital labor ratio in case of substitution).

3.3.2.2 The Solutions

As we have not considered dropping these series from the model, some computation has to be imagined. Considering our limited knowledge of the South African economy, its sources of information, and the limited time allowed, it is quite possible that better solutions can be found. The programs we will provide in the second part will allow users to apply their own ideas, in a way we will try to make as easy as possible.

For employment and wages, obviously this information is difficult to guess. Our research has led us to a data set provided by the International Labor Organization, which contains most of the information we need. The access is presented in the second part.

For housing investment, for the time being we have put it at 10% of total investment.

For capital, the only information we have is productive investment (actually including housing). To define capital, we must do the following:

- Decide on a starting value (before the first period at which investment is known, and hopefully GDP).
- Set a ratio between end of period capital and the next period GDP. A common value is 2.
- Decide on a depreciation rate. A common value is 0.05.
- Apply the identity computing end of period capital by the sum of the non-depreciated part of the previous value, and investment of the period.

More sophisticated methods are available, like the permanent inventory:

K = K(-1) * (1-rdep) +IP

For intermediate consumption, a rough guess will state that each unit of value added needs one unit of intermediary consumption. Of course, the ratio (the “technical coefficient”) changes a lot with products, but on the whole this is a reasonable assumption.

Non-wage revenue of households from production will be defined as a share of firms’ margins. In our opinion, the value could be close to 30%, but a better guess can certainly be made by local specialists. Of course the value is linked to wages, to provide a global share of household revenue.

Finally, social contributions will be measured as a share of wages. A value of 20% for both workers and employers looks reasonable, but again local specialists could do better.

3.3.2.3 Conclusion

Now that we have produced, more or less accurately, all the series we need, all we are lacking to have a full model is to replace the theoretical behaviors by actual formulas. This is the next step, but first we must check the following:

- That all the identities hold true using the present data.
- That all the series needed for estimation are present.
- This is the goal of a specific technique, the residual check, which we shall present in the second part.

To statistically determine productive capacity, we have several options depending on the available information.

In some countries (such as France), a survey asks firms by how much they could increase their production using the present factors (hiring more people if necessary). This gives the firm's capacity. Using the same weights as for computing actual production, one gets a comparable measure of capacity, and the rate of use as a ratio of the global values.

Then we shall use the capacity series to estimate its equation. For this, we can specify the actual behavior of firms, and optimize their profits under a capacity constraint using the formula we want to estimate. This applies when the factors are substitutable (otherwise the optimum solution is set from the start, and does not depend on relative costs). Taking the derivative of the function according to both labor and capital will give a set of equations with common coefficients, which one can estimate as a system. This method takes into account fully and explicitly the role of the relative costs.

If we know only the level of factors (capital is sometimes missing in the country's statistics), we can specify the production function, and estimate its parameters over the actual values of production. We can suppose that the estimated formula gives normal production, and the residual is the output gap. Again, the ratio of actual to "normal" production gives the rate of use, but this time to a constant factor (the average rate of use).

We can also (a better solution in our opinion) apply the first method, using actual production instead of capacity. Again, the estimated capacity (reconstructed by applying the production function to the estimated factors, considered as optimal) will give a normal level of production, and the difference to actual production the output gap.

If we do not have this information, we can always smooth production, and use the result as a "normal production" level (at a normal rate of use of capacities). For this, applying to actual data a Hodrick-Prescott filter is the most usual technique. If we suppose the "normal" rate of use of capacities constant over time, we get capacity at an unknown multiplicative factor.

This technique does not require a choice of production function, or the availability of a series for capital (which is often absent or unreliable). Neither does it provides it, which will be a problem for model specification.

3.4 The Third Task: Estimating the Equations

We will now consider in turn the behaviors we have described earlier, to which we will associate actual equations.

We will limit the elements related to EViews, which will be presented in the second part. First, however, we have to address a specific issue.

For a long time now, the study of the so-called "cointegration" of a set of variables has been introduced as an important criterion, if not a prerequisite, to establishing a behavioral link between these variables. We shall not try to describe fully this technique, but to give some elements to help understand its basis and purpose. The reader will find some useful references at the end of the volume.

We shall start with error correction models. Even earlier than cointegration, this type of model has been favored both by modellers and econometricians, as it presents a more consistent structure and allows a better interpretation of equation specifications. If the sample is large enough, it also allows for the relation of the estimation diagnosis to cointegration.

But first, we have to address the issue of stationarity.

3.4.1 Stationarity and the Dickey-Fuller Test

The basic issue of stationarity should not apply to estimated equations, as it considers a single variable or expression. However:

- Some estimated equations consider indeed a single variable!
- Considering stationarity is essential to cointegration, a multi-variable technique.

As we have seen earlier, the basic least squares estimation method requires the absence of autocorrelation of residuals, or rather a reasonable proof that this is true (such as a 95 percent probability). Now we shall consider the reverse situation: that the residual is perfectly correlated, or that the correlation coefficient is equal to 1. We shall apply this to the case of a variable with a constant mean:

$$x_t = a + e_t$$

$$e_t = \rho \cdot e_{t-1} + u_t$$

The question also will be different. We shall not consider an actual explanation (at least for the time being) but whether the variance of the process grows indefinitely, or is bounded in the long run.

The logic used is quite simple. Let us call σ^2 the variance from the period (the error is homoscedastic). Starting with period 1, the variance of the error after T periods will be:

$$V_T = \sigma^2 + \rho^{2\cdot\cdot} \cdot \sigma^2 + + \rho^{4\cdot\cdot} \cdot \sigma^2 + \ldots\ldots = \sum_{t=1}^{T} \rho^{2\cdot(T-t)\cdot} \sigma^2$$

If ρ=1, the error which appears at a given period is transmitted fully to the next one, the total variance after T periods will be the sum of individual ones, or T times the single period variance σ^2. It will grow to infinity with t.

$$V_T = \sum_{t=1}^{T} 1^{2\cdot(T-t)\cdot} v_t = \sum_{t=1}^{T} v_t = T \cdot \sigma^2$$

On the contrary, if ρ<1, it is reduced from one period to the other (in proportion $\rho^2 < 1$), and the global variance V reached after t periods will be the sum of t individual variances v:

$$V_T = \sum_{t=1}^{T} \rho^{2\cdot(T-t)\cdot} v_t = \sum_{t=1}^{T} \rho^{2\cdot(T-t)\cdot} \cdot \sigma^2 = \sigma^2 \cdot \sum_{t=1}^{T} \rho^{2(T-t)\cdot} = \sigma^2 (1-\rho^{2T\cdot})/(1-\rho^{2\cdot})$$

which converges to $\sigma^2/(1-\rho^{2\cdot})$ when t go to infinite.

In other words, when we move from period t to t+1, the change in the variance is the addition of an element from t+1 periods back. This term goes to zero when t grows to infinity, provided it is reduced with time.

If ρ is lower than 1, we call the series stationary (if not, non-stationary of course). The distance of actual values of x to its mean a is bounded.

Note: we do not consider ρ >1, as this would bring an explosive process which does not correspond to economic reality.

To check for stationarity, the most usual tool is the Dickey Fuller test, which tries to establish if ρ -1 is lower than zero. For this, it regresses the time variation of the variable over its lagged value:

$$\Delta x_t = e_t - e_{t-1} = (\rho - 1) \cdot e_{t-1} + u_t$$

and compares the (normally negative) T-statistic of the explanatory variable to values in a table. Due to the presence of the lagged value on both sides of the equation, the test is more demanding, and the 5% probability requests a value of around 3.

The test can be "augmented" by introducing the change lagged one or more times as additional explanatory variables.

Now, what is the relation of stationarity with econometric estimation? We shall see later the application to cointegration, but we shall start with a simpler case: single variable estimation, or rather the estimation of the evolution of the distance to a given target.

Using the above framework, this means that we consider "a" as a target, and x as a variable stationary around that target (which means that the distance to the target is stationary or bounded).

Another way of considering the problem is to build from the variable (say imports) an expression which should be stationary (like its share in total demand) and to look for the stationarity of that concept.

We want to see if the ratio is stationary, with:

$$Log(M_t / TD_t) = a + e_t$$

3.4.1.1 A First Error Correction Model

Until now we have considered that the process applied to a single element, with a constant target. The only correction could come from the residual. If this element is the ratio of imports to local demand, we have:

$$e_t = Log(M_t / FD_t) - a = \rho \cdot e_{t-1} + u_t = \rho \cdot [Log(M_{t-1} / FD_{t-1}) - a] + u_t$$

We can suppose that the correction process will be driven, not by the ratio itself, but by one element, imports in our case. Then the same process will be formulated as:

$$Log(M_t) = Log(FD_t) + a + \rho \cdot (Log(M_{t-1} / FD_{t-1}) - a) + u_t$$

$$\Delta Log(M_t) = \Delta Log(FD_t) - (1-\rho) \cdot (Log(M_{t-1} / FD_{t-1}) - a) + u_t$$

This means that a change in FD is followed by the same relative change in M, and that the only change in the ratio comes from:

- the residual u(t)
- correcting the gap between the ratio and a, in the proportion (1-ρ)

But a change in final demand need not be followed immediately by a proportional change in imports. We shall have in this case:

$$\Delta Log(M_t) = \alpha \cdot \Delta Log(FD_t) - (1-\rho) \cdot (Log(M_{t-1} / FD_{t-1}) - a) + u_t$$

What we have here is the simplest form of an error correction model.

- As in the previous case ,the process corrects the previous gap to the target, and introduces a new gap through the new residual,
- But the non-identical[14] adaptation of one element to the other introduces a new source of gap

If α=1 we shall speak of “dynamic homogeneity,” We shall see later the consequence of this assumption.

More generally, we can also consider

$$A(L) \cdot \Delta Log(M_t) = B(L) \cdot \Delta Log(FD_t) - (1-\rho) \cdot (Log(M_{t-1} / FD_{t-1}) - a) + u_t$$

$$A(L) \cdot \Delta Log(M_t) = B(L) \cdot \Delta Log(FD_t) - (1-\rho) \cdot (Log(M_{t-1} / FD_{t-1}) - a) + u_t$$

[14] With α>1, the change in M will overshoot the change in FD, producing a gap with the opposite sign. This does not contradict the error-correcting capacity of the process.

where A and B are lag polynomials matrixes.[15]

The simplest form is:

$$\Delta Log(M_t) = \alpha \cdot \Delta Log(FD_t) - \beta \cdot (Log(M_{t-1} / FD_{t-1}) - a) + u_t$$

The interpretation is easy. The agent has a long-term target, a formulation depending on one or several explanatory terms.

- If the target changes, he will adapt partially the actual value, in proportion α.
- If at the previous period, there was a gap between the desired and actual ratios, he will close it partially, in proportion β.

The values for α and β should be positive (for the general matrix formulation, the dynamic term should lead in the right direction). In theory the coefficient α can have any (positive) size, but a value higher than one supposes some initial overshooting. As to β, its value should also be lower than one. Between 1 and 2, it will create an overshooting but convergent process. Higher than two, it will produce a divergence.

If these conditions are met, the gap produced by a shock on the target at a given period will close regularly, in proportion α at for first period, and β = (1-ρ) for the next ones.

This also guarantees the long-term convergence of x_t to a steady state growth path, if explanatory elements do. Let us get back to a formulation in logarithms.

$$\Delta Log(x_t) = \alpha \cdot \Delta Log(x_t^*) - \beta \cdot Log(x_{t-1} / x_{t-1}^*) + \gamma + u_t$$

This can be written as

$$Log(x_t) = (1-\beta) \cdot Log(x_{t-1}) + \alpha \cdot \Delta Log(x_t^*) + \beta \cdot Log(x_{t-1}^*) + \gamma + u_t$$

If x* reaches a steady state growth path (with a constant growth rate q), we have

$$Log(x_t) = (1-\beta) \cdot Log(x_{t-1}) + \alpha \cdot q + \beta \cdot (q \cdot (t-1) + d) + \gamma + u_t$$

Derivation of the expression gives

$$\Delta Log(x_t) = (1-\beta) \cdot \Delta Log(x_{t-1}) + \beta \cdot q + \Delta u_t$$

This means that the growth rate of x will converge to q, provided β is lower than one (or even than two, if we accept alternate convergence).

[15] Let us recall the use of these polynomials.

If $A(L) = a + b \cdot L + c \cdot L^2$

then

$A(L) \cdot x_t = a \cdot x_t + b \cdot x_{t-1} + c \cdot x_{t-2}$

The gap between x and x* will converge to

$$q = \alpha \cdot q + \beta \cdot (Log(x^*_{t-1}) - Log(x_{t-1}))$$

or

$$Log(x_t) - Log(x^*) = (1-\alpha) \cdot q / \beta$$

We can observe the gap will not depend on starting values.

It will be zero only if

a = 1, a case we have already called "dynamic homogeneity".

q = 0

These reasons are easily explained:

- If a=1, any change in the target will be closed immediately. As the error correction process closes the initial gap with time, the global error will converge to zero.
- If q=0, the target does not move. And again, the error correction process closes the initial gap with time.

If none of these conditions is met, a long term gap will build up, the higher with

- Small values of a and b
- High values of q.

3.4.1.2 The Interest of this Formulation

In a model, using an error correction model structure can follow several goals:

- **Controlling the stability of formulations.** Numreic values of coefficients immediately tell if the equations give stable solutions.
- **Improving the stability of numerical properties.** In particular, the dynamic correcting mechanisms will dampen with time the effects of estimation errors on past periods and of the random term.[16]
- **Identifying immediately the long term formulation.** The long-term model can be produced directly by extracting the error definition from the full formulation[17,18].

[16] But not of the error on the coefficient, unfortunately.
[17]Deleau M, Malgrange P, *méthodes d'analyse des modèles empiriques*, Annales de l'INSEE no [20], septembre 1975
[18]Brillet JL: *Propriétés de long terme de la maquette Micro-DMS*, in "Biologie et Economie, les apports de la Modélisation", J Demongeot et P Malgrange editeurs, 1988

- **Allowing a better understanding of behaviors.** Regarding sensitivities of the model path to shocks on assumptions, it will be easier to separate the contributions of permanent changes in the target and the dynamics which lead back to them.

A few caveats:

- It can be considered abusive to speak of error corrections. The actual number of observations a macro economic model has access to is often very limited (especially for an annual one). The precision of estimation in the long-term target formulation might be too low for the gap to be called "error."
- The size of the sample must be high to be allowed to apply this kind of tests, although one is tempted to do so. But their conclusions might still be useful, even if not truly accurate.

3.4.1.3 Cointegration

Error correction models have been present in economic theory, and modelling in particular, for quite a long time, actually long before the name was invented. For instance, investment equations including the rate of use of productive capacity as an explanatory variable, or adapting employment to a structural productivity target, both actually follow an error correction framework. But it is only in the last thirty years that error correction equations have generalized, and specific econometric methods have been designed, linked to the ECM framework. These methods fall under the general concept of *cointegration*, which we shall summarize now, trying to avoid as much as possible too technical concepts.

Let us consider a single time – varying variable, its value at a given period being the sum of a constant term and a random term depending on its previous value. We have seen earlier that its variance can be bounded, making it *stationary*. If it is not, one can compute successive time differences of the variable. If the nth time difference becomes stationary, the variable will be said to be integrated of order n, in short I(n).

But it is also possible that between a set of non-stationary variables (or expressions), one can evidence a linear combination, presenting the characteristics of stationarity. In other words, the impact of any difference of this expression to a given (constant) target, is reduced at the next period, making the new, composite variable stationary (or I(0)). The relation is considered as “cointegrating”. Of course, with a set of variables, several cointegrating relations can appear, the maximum number being that of elements in the set.

Typical examples of this type of long run relationship could apply to:

- The share of wages in values added and the unemployment rate.
- The share of exports in world demand and price competitiveness of exports, or/and the rate of use of productive capacities.

- The ratio of labor to capital and their relative costs.

These elements will be non-stationary individually, but they will be linked in the long run: if the long run value of one changes, the other will change in a given proportion.

In our previous ECM formulation, expressed here in levels:

$$\Delta x_t = \alpha \cdot \Delta x_t^* + \beta \cdot (x_{t-1}^* - x_{t-1}) + \gamma + u_t$$

The cointegrating relation is of course:

$$x_t - x_t^* = 0$$

But this time x_t^* is a formula which should contain estimated coefficients.

The estimation of the error correction formulation cannot be done directly (using nonlinear least squares). One should first test the existence of the cointegrating relation, using a specific test. Then, if (and only if) a cointegration equation has been evidenced, one should proceed to the estimation of the dynamic error correction formula, using the cointegrating residual as the error term.

Of course, the coefficient in the cointegrating equation must be considered acceptable, both from a statistical point of view (using the T-statistic like in a normal regression) and an economic one (the elasticity of the share of imports in demand must be negatively influenced by the ratio of import to local deflators). In the following examples, we shall address this process twice.

Unfortunately, one drawback of this framework is its requirements in terms of sample size (fifty observations at the very least). This means in particular that cointegration should not be considered for yearly macroeconomic models: either the sample is too small, or its span (fifty years) makes quite dubious the production of time-consistent formulas. In particular, the separation of variables at current prices into elements at constant prices and deflators loses much of its accuracy decades from the base year.

This explains certainly the fact that most operational models use two step cointegration quite sparingly. Very often an error correction framework is indeed introduced (if only to profit from the advantages above), but estimation will be done in one step. This can be acceptable for small samples, but introduces an inconsistency, as the results will be different from the two step method (otherwise cointegration would probably have worked).

3.4.2 The Production Block

We shall now consider our first behavior, the one which we meet at the beginning of any model description. This part of the model (one speaks often of "blocks") will not define production, but rather potential production (or productive capacity), as a function of available factors.

Why not actual production itself? There are two ways to consider production:

- Actual local production, contributing with foreign exporters to the satisfaction of demand (both local and foreign) demand, in a share depending on relative prices and available capacities).
- Potential production, given by the production function, taking into account the level of factors (capital and labor), themselves chosen by firms according to their relative costs, expected demand, and profits conditions.

We want our model to follow the most logical causal sequence, which is:

- Defining target capacity depending on profit conditions and expected demand.
- Choosing the optimal level of factors allowing this capacity.
- The actual levels will adapt, giving potential production.
- Global demand will follow, and will be shared between local and foreign producers to give actual production.
- Imperfect knowledge of future demand, technical difficulties, and concerns in a fast adaptation of factors will contribute to the creation of a gap between potential and actual value.

The comparison between actual and potential production will play an important role in some behaviors.

This is the sequence that the model will describe, actual production being obtained late in the process, once demand is known (as in the small model).

This capacity for production will be measured:

- For employment, in years or quarters according to model periodicity.
- For capital, at constant prices, in the currency of the country.

The function can also include:

- Energy consumption
- Intermediate goods (like raw materials).

Actually, capacities are generally defined in terms of value added, a more reliable notion as we have explained earlier. This means the two last elements are not taken into account, or rather their level will come automatically from value added itself.

The first issue concerns the logical link between capacity and factors. We have already seen:

- **Complementary factors.** For a given capacity, there is a single optimal process using a fixed combination of labor and capital. Starting from an optimal combination, adding to the process a quantity of one factor does not increase capacity, or allow using less of the other factor. This capacity is obviously optimal regardless of the relative costs. Actually labor productivity has generally some flexibility, and capital is the truly constraining factor, as temporary and limited increases in labor productivity can be achieved (for instance by increasing the number of hours worked).

This is the simplest option, in its formulation, estimation and understanding of properties. Operational models generally use more sophisticated frameworks:

- **Cobb-Douglas.** The elasticity of substitution is unitary. This means that if the ratio of the cost of labor to capital changes by 1%, the optimal ratio of capital to labor will change by 1% too, for a given capacity requirement.
- **CES (Constant Elasticity of Substitution).** Now the elasticity can take any fixed value (with the right sign).

Of course, the CES option covers both others (with fixed elasticities of 0 and 1, respectively).

The framework calls also for:

- **A definition of the relative cost.** The relative cost of labor and capital is not just measured by the ratio of the wage rate to the investment deflator. One has to take also into account:
    - Social contributions of firms: they contribute to the cost of labor.
    - The interest rate: while capital is bought immediately[19], labor can be bought (rented) when the time comes (slavery has been abolished for some time now). So a firm which has money can save it, and one which has not does not have to borrow.
    - The depreciation rate: capital wears out, while when a worker "wears out" through old age or sickness, he will leave and can be replaced by a new one at no cost except training (pensions have already been saved as a share of wages).
    - The future evolution of wages: if wages are currently growing faster than inflation, firms can expect labor to become less competitive. The gain from having output transferred to fast developing countries becomes lower as they close the gap with developed ones. This applies in particular to present China.
- **The possible changes in technology.** The issue here is to decide if the technology decided at investment time (which defines the roles of labor and capital) can change later.

Basically, the options are:

- A single available technology (Clay-Clay).
- A technology chosen at installation time, with no later change (Putty-Clay). This means basically that the "complementary factors" option applies to factors once they are installed.

[19] Actually some forms of capital (like buildings, computers or patents) can be rented or leased.

- A technology with a permanent possibility of change (Putty - Putty). The same substitution option applies to factors at any period.

3.4.2.1 A Specific Problem: the Statistical Determination of Productive Capacity

To determine capacity, we have several options, depending on the available information. In some countries (such as France), a survey asks firms by how much they could increase their production using the present factors (hiring more people if necessary). This gives the firm's capacity. Using the same weights as for computing actual production, one gets a comparable measure of capacity, and the rate of use as a ratio of the global values.

Then, we shall use the capacity series to estimate its equation. For this, we can specify the actual behavior of firms, and optimize their profits under a capacity constraint using the formula we want to estimate. This applies when the factors are substitutable (otherwise the optimum solution is set from the start, and does not depend on relative costs). Taking the derivative of the function according to both labor and capital will give a set of equations with common coefficients, which one can estimate as a system. This method takes into account fully and explicitly the role of the relative costs.

If we know only the level of factors (capital is sometimes missing in the country's statistics), we can specify the production function, and estimate its parameters over the actual values of production. We can suppose that the estimated formula gives normal production, and the residual is the output gap. Again, the ratio of actual to "normal" production gives the rate of use, but this time to a constant factor (the average rate of use).

We can also (a better solution in our opinion) apply the first method, using actual production instead of capacity. Again, the estimated capacity (reconstructed by applying the production function to the estimated factors, considered as optimal) will give a normal level of production, and the difference to actual production the output gap.

If we do not have this information, we can always smooth production and use the result as a "normal production" level (at a normal rate of use of capacities). For this, applying to actual data a Hodrick-Prescott filter is the most usual technique. If we suppose the "normal" rate of use of capacities constant over time, we get capacity at an unknown multiplicative factor.

This technique does not require a choice of production function, or the availability of a series for capital (which is often absent or unreliable). Neither does it provides it, which will be a problem for model specification.

3.4.2.2 Productive Capacity

In this simple model, we shall consider a Cobb-Douglas framework:

$$Log(CAP(t)) = a + b.t + c.Log(LF(t)) + (1 - c).Log(K(t\text{-}1))$$

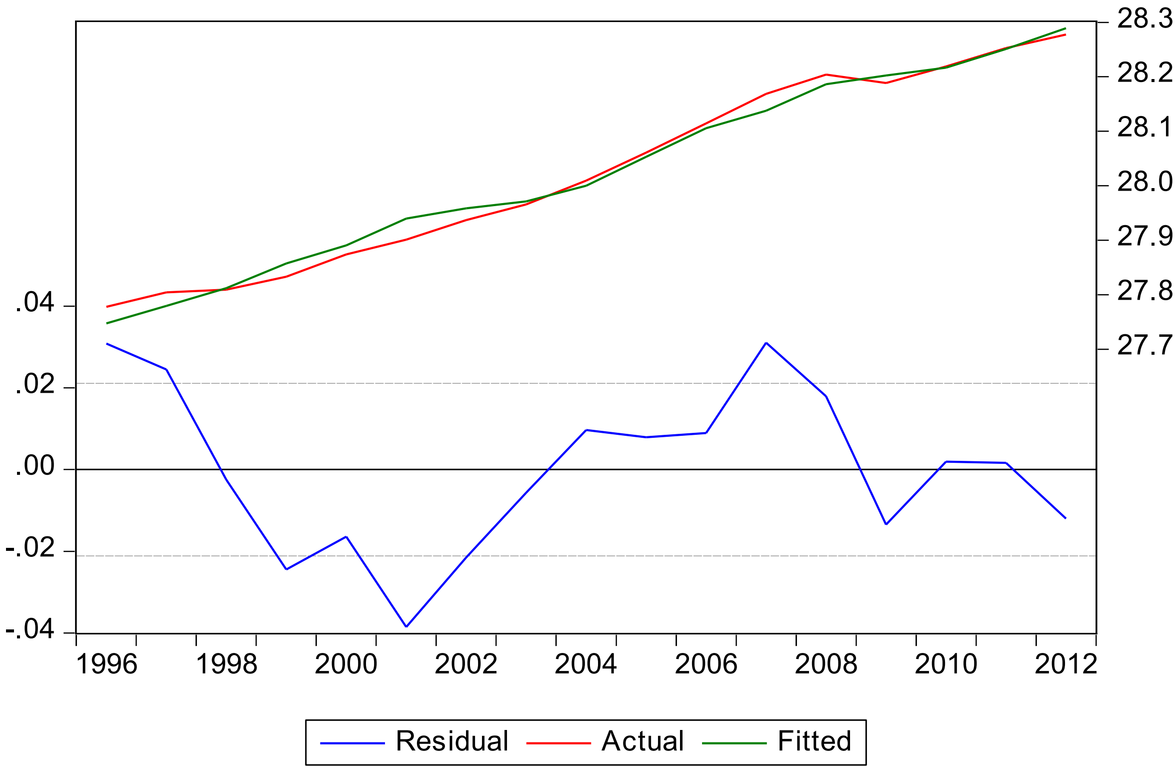
We shall suppose that estimation of the formula using Q (value added) as the explained variable allows to separate the capacity (the estimated element) from the gap between actual production and capacity (actually « normal production » or the level of production associated with a normal use of factors).

Again, it is clear that the size of the sample is quite small, and that the whole estimation process is somewhat questionable.

Dependent Variable: LOG(Q)
Method: Least Squares
Date: 04/01/14 Time: 13:24
Sample: 1996 2012
Included observations: 17
LOG(Q)=(1-C_CAP(1))*LOG(K(-1))+C_CAP(1)*LOG(LF)+C_CAP(2)
+C_CAP(43*(T-2012)*(T<=2012)+C_CAP(1)*LOG((1+TXQ)/(1+TXN))
(T-2012)(T>=2012)

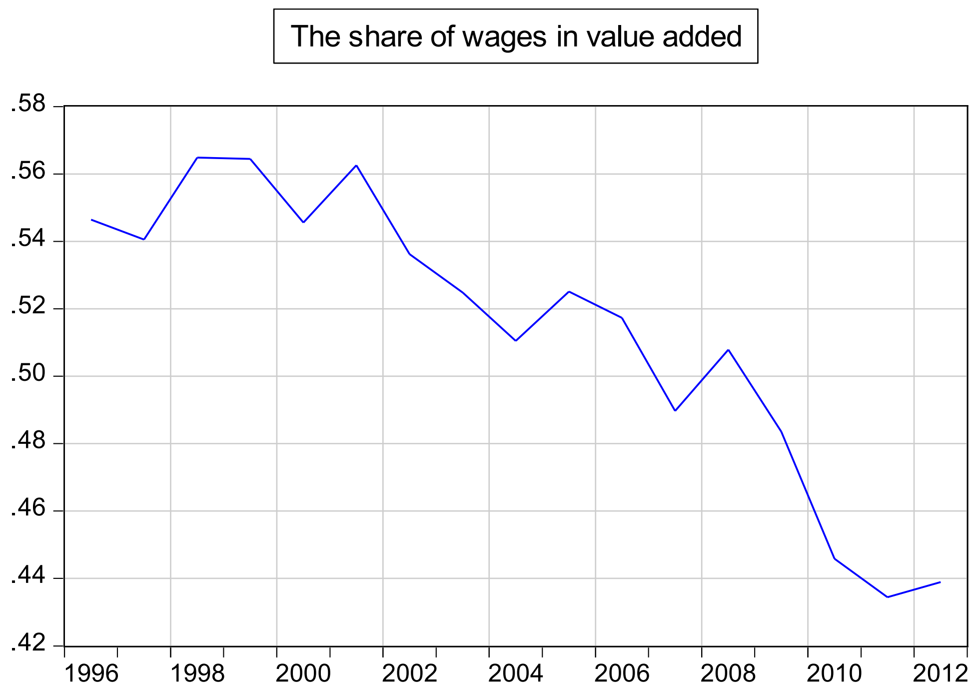
	Coefficient	Std. Error	t-Statistic	Prob.
C_CAP(1)	0.538464	0.105472	5.105278	0.0002
C_CAP(2)	6.202348	1.298467	4.776669	0.0003
C_CAP(3)	0.016571	0.001608	10.30347	0.0000

R-squared	0.987096	Mean dependent var	28.02351
Adjusted R-squared	0.985253	S.D. dependent var	0.174025
S.E. of regression	0.021133	Akaike info criterion	-4.717132
Sum squared resid	0.006253	Schwarz criterion	-4.570095
Log likelihood	43.09562	Hannan-Quinn criter.	-4.702516
F-statistic	535.4657	Durbin-Watson stat	0.744547
Prob(F-statistic)	0.000000		

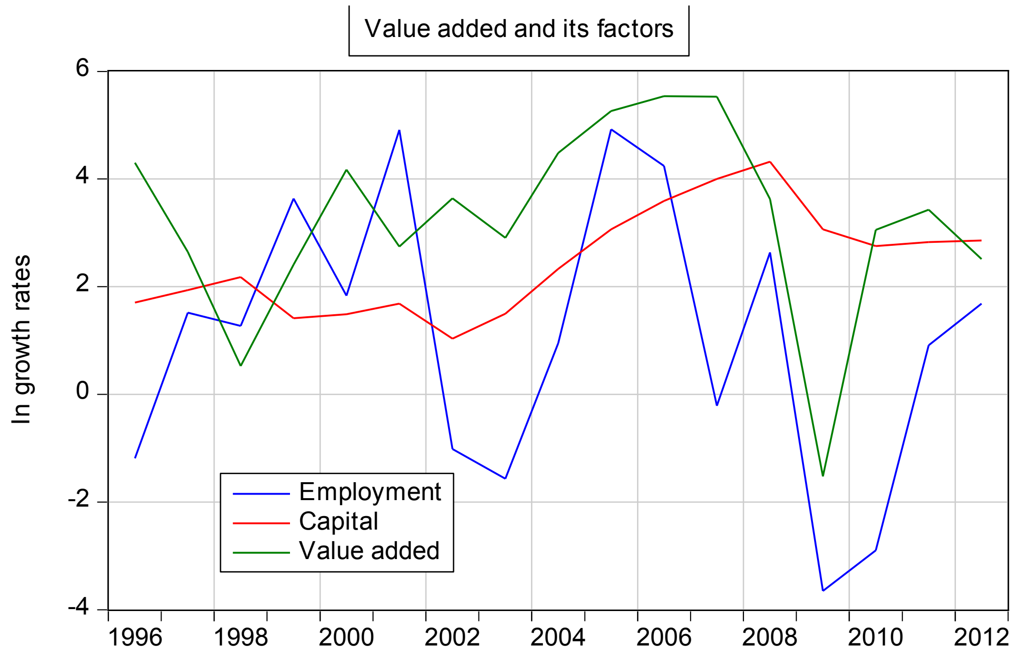


We can see that the fit is quite good, which was to be expected as value added, labor and capital have shown a relatively stable growth over the estimation period. What is more interesting is that estimation separates clearly from the trend the role of the factors, and that their contribution follows an usual share, perhaps a little lower for employment than usual. As could be expected, this value is not too far from the share of wages in value added.

One will note that in this case the Durbin-Watson test is irrelevant: what we try to estimate is not a behavior explaining actual value added (it will be defined as balancing the supply-demand equilibrium) but the underlying unknown capacity, interpreting the difference to the actual value as the output gap. This gap does not have to be temporally uncorrelated. Indeed considering the inertia of factor adaptation (in particular capital) we do expect some correlation, which must not be too strong however. Observing the above residuals, we might observe (with a little imagination) a cycle with 5 year half periods, a reasonable value.



In terms of growth rates, we can make out a link between factors and output.



Finally, one should not question the low value of the Durbin-Watson test: it just shows the presence of cycles, with a reasonable period.

3.4.2.3 Productive Investment

This basic economic idea is quite simple. The purpose of investment is:

- To replace discarded capital.
- To allow a higher level production, facing an increase of demand.
- To allow capacity to adapt to a given ratio of production.

Taking into account the evolution of capital productivity.

Let us first consider capital. To adapt to this implicit target, its growth rate must be the sum of three components:

- The expected growth rate of value added, if capital productivity is stable and the rate of use is optimal.
- The growth rate which leads to the optimum if this is not the case.
- Corrected negatively by the evolution of capital productivity.

In addition, investment must also compensate depreciation. The equation for capital

$$K_t = K_{t-1} \cdot (1 - dr_t) + I_t$$

can be written as

$$tx(K_t) = -dr_t + I_t / K_{t-1}$$

This gives us

$$I_t^* / K_{t-1} = tx(K_t) = dr_t + tx^*(K_t) = dr_t + tx^a(Q_t) - tx^*(UR_t) - tx(pk_t)$$

with a rate of use given by the ratio between value added and the estimated result of the Cobb Douglas function.[20]

In other words:

- If firms expect a growth rate of 4%, capacities should adapt to that growth.
- But if they feel their capacities are under-used by 1%, their desired capacity will only increase by 3%.
- If capital productivity is going to increase by 1%, they will need 1% less capital.
- But once capital growth has been defined, they also have to compensate for depreciation.

If we suppose that the depreciation rate is constant, as well as the rate of growth of capital productivity; production growth expectations are based on an average of the previous rates; and the rate of use the ratio of actual GDP to a value obtained under normal utilization of factors, which leads to a unitary target, we get the simplified formula:

$$I_t^* / K_{t-1} = a + \sum_{i=0}^{n} \alpha_i \cdot tx^a(Q_{t-i}) - tx^*(UR_{t+1})$$

With

$$\sum_{i=0}^{n} \alpha_i = 1$$

Finally, we can suppose, as we shall do also for employment, that the desired growth of capital is only partially reached in practice, either because firms react cautiously to fluctuations of demand, or because they are constrained by investment programs covering more than one period.

And we shall leave free the coefficients:

$$I_t^* / K_{t-1} = b \cdot I_{t-1} / K_{t-2} + (1-b) \cdot (a + c \cdot \sum_{i=0}^{n} \alpha_i \cdot tx^a(Q_{t-i}) - d \cdot tx^*(UR_t))$$

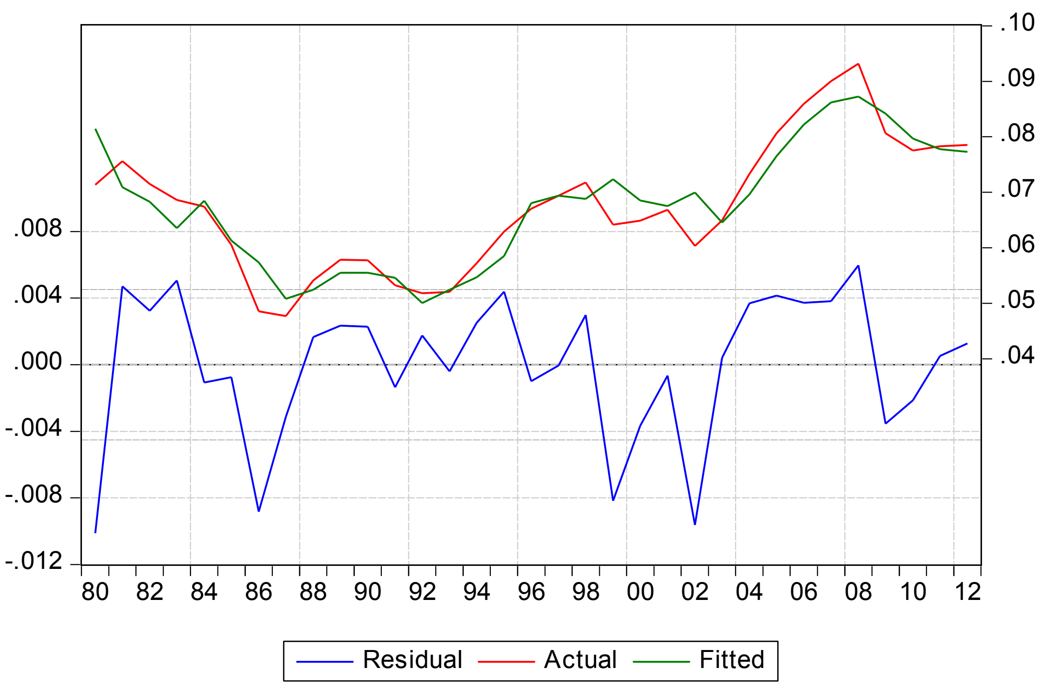
[20] This value represents maximum capacity multiplied by the optimal rate of use (supposed constant).

We get the following result:

Dependent Variable: IP/K(-1)
Method: Least Squares
Date: 04/01/14 Time: 17:33
Sample: 1980 2012
Included observations: 33
IP/K(-1)=C_IP(1)*IP(-1)/K(-2)+C_IP(2)*@PCH(Q)+C_IP(3)*LOG(UR)
+C_IP(4)

	Coefficient	Std. Error	t-Statistic	Prob.
C_IP(1)	0.735156	0.078037	9.420619	0.0000
C_IP(2)	0.100418	0.035810	2.804200	0.0089
C_IP(3)	0.006225	0.002845	2.188112	0.0369
C_IP(4)	0.017295	0.005931	2.916143	0.0068

R-squared	0.865791	Mean dependent var	0.067452
Adjusted R-squared	0.851907	S.D. dependent var	0.011733
S.E. of regression	0.004515	Akaike info criterion	-7.849466
Sum squared resid	0.000591	Schwarz criterion	-7.668071
Log likelihood	133.5162	Hannan-Quinn criter.	-7.788432
F-statistic	62.36022	Durbin-Watson stat	1.544535
Prob(F-statistic)	0.000000		



The coefficients are all significant and their value reasonable, except for the rate of use which show a very low value.

The equation we have built is not only satisfactory by itself, but we can expect it to provide the model with adequate properties. In particular, the long term elasticity of capital to production is now unitary by construction, provided employment adapts also. Starting from a base simulation, a 1% permanent shock on Q will leave the long run value of UR unchanged[21]. This gives the same relative variations to production, capacity and (with a constant capital productivity) capital.

The coefficients "a" and "b" determine only the dynamics of the convergence to this target.

We have actually estimated a kind of error-correction equation, in which the error is the gap between actual and target capacity (the rate of use).

We hope to have made clear that to produce a consistent formulation, in particular in a modelling context, one must start by establishing a sound economic background.

3.4.2.4 Employment: Stationarity, Error Correction Models, Breakpoint Test.

The employment equation should follow also a complementary factors framework.

In the previous paragraph, we showed that in this framework the element determining capacity is the sole capital; while firms could ask from workers a temporary increase in productivity, high enough to ensure the needed level of production[22]. Adapting employment to the level required to obtain a "normal' productivity target will be done by steps.

This means estimating employment will allow us to apply the elements on error correction models we have presented earlier, in a very simple framework.

We shall suppose that firms:

- Know the level of production they have to achieve.
- Know also the level of production which should be achieved by each worker under normal circumstances (in other term his normal productivity).

From these two elements they can determine the normal number of workers they need.

But they do not adapt the actual employment level to this target, and this for:

- **Technical reasons**. Between the conclusion that more employees are needed and the actual hiring[23], firms have to decide on the type of jobs called for, set up their demands, conduct

---

[21] As the left hand side represents the (fixed) long term growth rate of capital.

[22] This is true in our macroeconomic framework, in which the changes in production are limited, and part of growth is compensated by increases in structural productivity (due for instance to more capital intensive processes). At the firm level, employment can produce bottlenecks. This will be the case if a sudden fashion appears for particular goods requiring specialized craftspeople, even if the tools and machines are available for buying.

[23] But not the start of actual work: what we measure is the number of workers employed, even if they are still training for instance.

interviews, negotiate wages, establish contracts, get authorizations if they are foreign citizens, maybe ask prospective workers to train... Of course this delay depends heavily on the type of job. And this goes also for laying out workers.

- **Behavioral reasons**. If facing a hike in production, firms adapt immediately their employment level to a higher target, they might be faced later with over employment if the hike is only temporary. The workers they have trained, maybe at a high cost, have no usefulness at the time they become potentially efficient. And laying them out will call generally for compensations.

"Normal" labor productivity does not depend on economic conditions. It might follow a constant trend over the period, such as:

$$Log(pl_t) = a + b \cdot t$$

Firms use this target to define "normal" employment:

$$LE_t^* = Q_t / pl_t^*$$

They adapt actual employment to this target with some inertia:

$$\Delta Log(L_t) = \alpha \cdot \Delta Log(L_t^*) + \beta \cdot Log(L_{t-1}^* / L_{t-1}) + \gamma + \varepsilon_t$$

We recognize here the error correction framework presented earlier, which requires:
$Log(L_t^* / L_t)$ to be stationary.

But α does not have to be unitary. However, if we follow the above reasoning, its value should be between 0 and 1, and probably significantly far from each of these bounds.

To estimate this system we face an obvious problem: pl* is not an actual series (LE* either, but if we know one we know the other).

But if we call "pl" the actual level of productivity (Q/LE) we can observe that:

$$Log(L_t^* / L_t) = Log((Q_t / pl_t^*)/(Q_t / pl_t)) = -Log(pl_t^* / pl_t)$$

The stationarity of $Log(L_t^* / L_t)$ is equivalent to that of $Log(pl_t^* / pl_t)$

Now it should be obvious that if pl* and pl have a trend, it must be the same, actually the trend defining completely pl*. If not, they will diverge over the long run, and we will face infinite under or over employment. So target productivity can be identified using the trend in the actual value, if it exists.

This means we can test the stationarity of the ratio as the stationarity of actual productivity around a trend, a test provided directly by EViews.

We can expect a framework in which actual productivity fluctuates around a regularly growing target, with cycles which we do not expect to be too long, but can last for several periods[24].

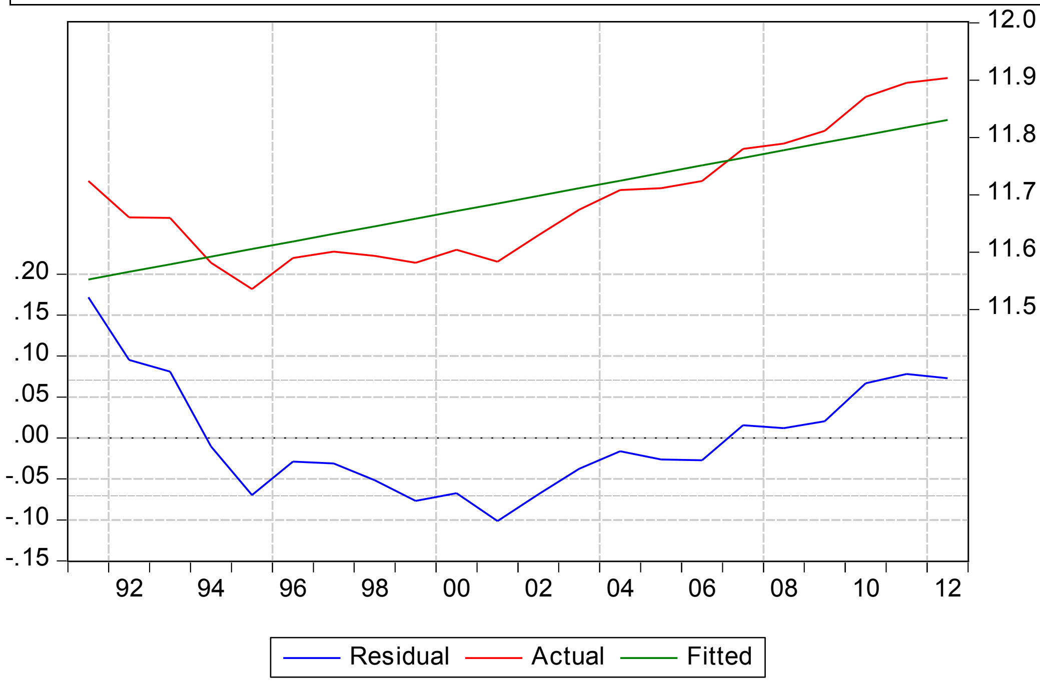
3.4.2.4.1 The First Estimations

We regress labor productivity on a time trend to get structural productivity.

Dependent Variable: LOG(Q/LF)
Method: Least Squares
Date: 04/02/14 Time: 10:42
Sample (adjusted): 1991 2012
Included observations: 22 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-14.83910	4.749017	-3.124668	0.0053
T	0.013255	0.002373	5.586563	0.0000

R-squared	0.609449	Mean dependent var	11.69145
Adjusted R-squared	0.589921	S.D. dependent var	0.110257
S.E. of regression	0.070606	Akaike info criterion	-2.376901
Sum squared resid	0.099704	Schwarz criterion	-2.277716
Log likelihood	28.14592	Hannan-Quinn criter.	-2.353536
F-statistic	31.20968	Durbin-Watson stat	0.288111
Prob(F-statistic)	0.000018		



[24] Which will create (acceptable) autocorrelation in the difference to the trend.

Results are quite bad. Productivity shows significant growth, but the standard error is quite high (more than 5 %). More important, the graph of residuals and the auto-correlation test show that we are not meeting the condition we have set: that observed productivity fluctuates around a trend, with potential but not unreasonably long cycles.

The problem apparently lies in the fact that the average growth rate is consistently higher in the first part of the period, and lower later. Seen individually, each sub-period might seem to meet the above condition.

From the graph, we clearly need at least one break.

For choosing the most appropriate dates, we can use two methods:

- A visual one: 2001 could be chosen, possibly plus or minus 1 year.
- A statistical one: the most appropriate test is the Chow Breakpoint Test.

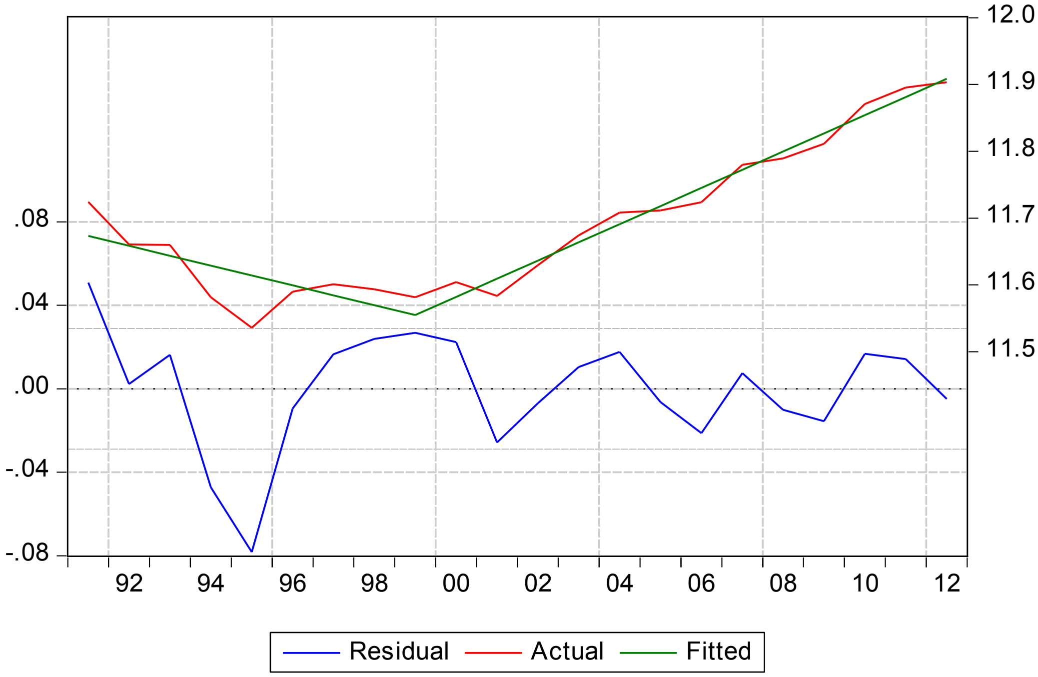
This test accepts the presence of a break around 2000, the best statistical result (the maximum likelihood) being obtained for 1999.

We will introduce the break in the estimation.

Dependent Variable: LOG(Q/LF)
Method: Least Squares
Date: 04/02/14 Time: 10:54
Sample (adjusted): 1991 2012
Included observations: 22 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-42.82868	3.405785	-12.57528	0.0000
T	0.027205	0.001699	16.01443	0.0000
(T-1999)*(T<=1999)	-0.042016	0.004197	-10.01147	0.0000

R-squared	0.937763	Mean dependent var	11.69145
Adjusted R-squared	0.931212	S.D. dependent var	0.110257
S.E. of regression	0.028918	Akaike info criterion	-4.122604
Sum squared resid	0.015888	Schwarz criterion	-3.973826
Log likelihood	48.34864	Hannan-Quinn criter.	-4.087556
F-statistic	143.1427	Durbin-Watson stat	1.222894
Prob(F-statistic)	0.000000		



One will note:

- that we have introduced no residual,
- we have introduced reversed trends, which stop after a while instead of starting inside the period, and
- the global trend starts in 2012.

This will be explained later.

The results look acceptable, as to the validation of coefficients and the graphs (we are presenting the program version, as the equation will be introduced in the model).[25]

[25] This is not absolutely needed, as a variable depending only on time can be considered exogenous and computed outside the model. But we want to be able to change the assumption in forecasts, and this is the easiest way.

Now we must test the stationarity of the residual. We shall use the Dickey-Fuller test (or Phillips – Perron).

Null Hypothesis: RES has a unit root

Exogenous: Constant

Lag Length: 1 (Automatic - based on SIC, maxlag=4)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.428744	0.0221
Test critical values:	1% level	-3.808546	
	5% level	-3.020686	
	10% level	-2.650413	

*MacKinnon (1996) one-sided p-values.

It works: the probability of the non stationarity (a unit root) is low enough.

From the trend in productivity and value added, we can compute target employment and estimate the employment equation, using a dummy variable for the 1994-195 period which shows a high residual.

Dependent Variable: DLOG(LF)
Method: Least Squares
Date: 04/02/14 Time: 13:37
Sample (adjusted): 1992 2012
Included observations: 21 after adjustments
DLOG(LF)=C(1)*DLOG(LFD)+C(2)*LOG(LFD(-1)/LF(-1))+C(3)+C(4)
*((T=1994)+(T=1995))

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.659426	0.173727	3.795757	0.0014
C(2)	0.733183	0.127989	5.728504	0.0000
C(3)	0.001373	0.004259	0.322467	0.7510
C(4)	0.072796	0.012230	5.952194	0.0000

R-squared	0.841158	Mean dependent var	0.019749
Adjusted R-squared	0.813127	S.D. dependent var	0.034477
S.E. of regression	0.014904	Akaike info criterion	-5.404735
Sum squared resid	0.003776	Schwarz criterion	-5.205779
Log likelihood	60.74972	Hannan-Quinn criter.	-5.361557
F-statistic	30.00821	Durbin-Watson stat	1.813653
Prob(F-statistic)	0.000001		

Following the reasoning made earlier, c (3)--or rather c(3)/c(2)-- will represent the logarithm of the long term gap between the target employment and the level reached. This gap will be significant if both:

- Employment shows a trend (the target is moving), which means that GDP and target productivity show different trends.
- A difference between the growths of GDP and target productivity is not compensated immediately (the value of c(1) is different from one).

The second condition is clearly met, but not the first.

3.4.2.5 Change in Inventories

We shall use this simplest estimation to present the basic features of EViews estimation, and also stress the necessity for homoscedasticity.

Our formulation will suppose simply that firms desire a level of stocks proportional to their production (or GDP). For a particular producer, this should be true both for the goods he produces and for the ones he is going to use for production. For instance, a car manufacturer will allow for a given delay between production and sale (for instance three months, which will lead to an inventory level of 1/4th of annual production). And to be sure of the availability of intermediary goods (like steel, tires, electronic components and fuel for machines in this case) he will buy the necessary quantity (proportional to production) sometime in advance.

We shall suppose that firms have achieved, at the previous period, an inventory level IL representing a number of semesters of production:

$$IL_{t-1} = a \cdot Q_{t-1}$$

and they want to keep this level at the present period:

$$IL_t^* = a \cdot Q_t$$

$$IL_t^* = IL_t$$

Then the change in inventory will represent:

$$IC_t = (IL_t - IL_{t-1}) = a \cdot \Delta Q_t$$

This means that contrary to the general case this equation should not include a constant term. Its presence would call for a trend (and a constant) in the equation in levels, with no economic justification. It would also introduce a problem: adding a constant to an explanation in constant Euros would make the equation non-homogenous.

Even then, the equation faces a problem, concerning the residual: between 1963 and 2004, French GDP has been multiplied by 4. We can suppose the level of inventories too (maybe a little less with economies of scale and improved management techniques).

It is difficult to suppose that the unexplained part of the change in inventories is not affected by this evolution. As the variable grows, the error should grow. But to apply the method (OLS), we need the residual to have a constant standard error. Something must be done.

The simplest idea is to suppose that the error grows at the same rate as GDP, which means that if we measure the change in inventories in proportion to GDP, we should get a concept for which the error remains stable. Of course, we shall have to apply the same change to the right hand side, which becomes the relative change in GDP.

To avoid causality problems (for a given semester, demand for IC is partly satisfied by Q), we shall use the previous value of Q.

The equation becomes:

$$IC_t \,/\, Q_{t-1} = a \cdot \Delta Q_t \,/\, Q_{t-1}$$

Finally, we can suppose that firms take time in changing their behavior:

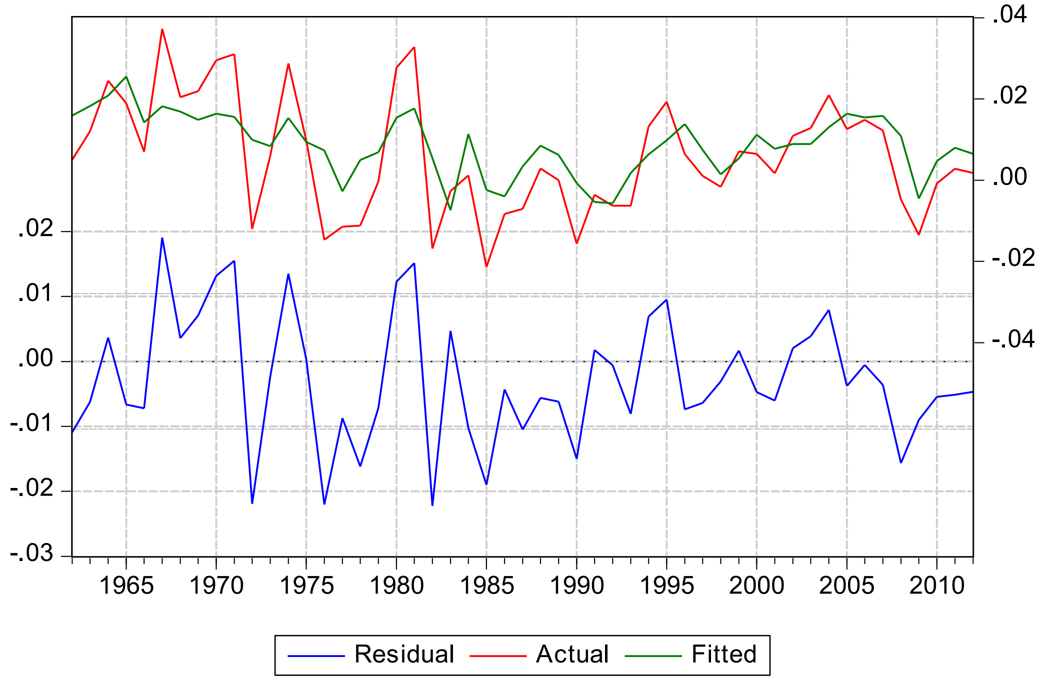
$$IC_t \,/\, Q_{t-1} = b \cdot a \cdot \Delta Q_t \,/\, Q_{t-1} + (1-b) \cdot IC_{t-1} \,/\, Q_{t-2}$$

This in turn give us:

Dependent Variable: IC/Q(-1)
Method: Least Squares
Date: 04/02/14 Time: 14:57
Sample (adjusted): 1962 2012
Included observations: 51 after adjustments
Convergence achieved after 1 iteration
IC/Q(-1)=C_IC(2)*C_IC(3)*@PCHY(Q)+(1-C_IC(2))*IC(-1)/Q(-2)

	Coefficient	Std. Error	t-Statistic	Prob.
C_IC(2)	0.812004	0.111376	7.290636	0.0000
C_IC(3)	0.290787	0.045507	6.390002	0.0000

R-squared	0.482953	Mean dependent var	0.005920
Adjusted R-squared	0.472401	S.D. dependent var	0.014346
S.E. of regression	0.010420	Akaike info criterion	-6.251675
Sum squared resid	0.005321	Schwarz criterion	-6.175917
Log likelihood	161.4177	Hannan-Quinn criter.	-6.222726
Durbin-Watson stat	1.648293		



According to the estimation, the sensitivity of inventories to an increase in value added is 0.29 x 0.81=0.23 in the short run and 0.29 in the long run, when the ratio stabilizes.

One will observe that we are using a very long period, perhaps too long. Considering only the last 20 years would give similar results.

Dependent Variable: IC/Q(-1)
Method: Least Squares
Date: 04/02/14 Time: 15:02
Sample: 1993 2012
Included observations: 20
Convergence achieved after 3 iterations
IC/Q(-1)=C_IC(2)*C_IC(3)*@PCHY(Q)+(1-C_IC(2))*IC(-1)/Q(-2)

	Coefficient	Std. Error	t-Statistic	Prob.
C_IC(2)	0.770647	0.202468	3.806265	0.0013
C_IC(3)	0.233535	0.055768	4.187639	0.0006

R-squared	0.452838	Mean dependent var	0.005852
Adjusted R-squared	0.422440	S.D. dependent var	0.008914
S.E. of regression	0.006774	Akaike info criterion	-7.056685
Sum squared resid	0.000826	Schwarz criterion	-6.957111
Log likelihood	72.56685	Hannan-Quinn criter.	-7.037247
Durbin-Watson stat	1.333172		

3.4.2.6 Unemployment

It actually seemed clearer to us to model the work force (employment + unemployment). As it depends on employment, the quality of estimation will be exactly the same (only the R-squared will change). The coefficient of the work force POPAC will be higher by 1 compared to a formulation using unemployment UN. Otherwise, the equation follows fully the framework defined above.

Note: we have divided all the elements by the exogenous population in age of working, to make the equation homogeneous and avoid heteroscedasticity.

The results are significant, but the values look high, especially the long term sensitivity to employment, which says that creating jobs does not really reduce unemployment, as most of them are taken by previously unemployed (or rather that a given job creation attracts to the labor market a similar number of persons).

Dependent Variable: D(POPAC)/POP1564(-1)
Method: Least Squares
Date: 04/02/14 Time: 14:43
Sample: 1996 2012
Included observations: 17
Convergence achieved after 31 iterations
D(POPAC)/POP1564(-1)=C_POPAC(1)*D(LT)/POP1564(-1)+C_POPAC(2)
D(POP1564)/POP1564(-1)-C_POPAC(3)(POPAC(-1)/POP1564(-1)
-C_POPAC(4)*LT(-1)/POP1564(-1)-C_POPAC(5))+POPAC_EC

	Coefficient	**Std. Error**	**t-Statistic**	**Prob.**
C_POPAC(1)	0.804607	0.165006	4.876226	0.0004
C_POPAC(2)	0.704220	0.243438	2.892807	0.0135
C_POPAC(3)	0.334596	0.114134	2.931610	0.0126
C_POPAC(4)	0.957881	0.427086	2.242830	0.0446
C_POPAC(5)	0.132911	0.171871	0.773318	0.4543

R-squared	0.856315	Mean dependent var	0.009671
Adjusted R-squared	0.808420	S.D. dependent var	0.013758
S.E. of regression	0.006022	Akaike info criterion	-7.146892
Sum squared resid	0.000435	Schwarz criterion	-6.901829
Log likelihood	65.74858	Hannan-Quinn criter.	-7.122532
Durbin-Watson stat	2.453489		

3.4.3 Prices

We shall now describe the behaviors associated with prices and wages.

3.4.3.1 The Value-Added Deflator

The formula will follow an error correction format linking the deflator to the wage cost (the yearly wage rate divided by the yearly productivity), with a long term unitary elasticity, defining the long term margin ratio as the long term explained variable, with a potential tradeoff with the rate of use (the firms apply the price which maximizes the value of output, taking into account the relationship between quantities sold and their price).

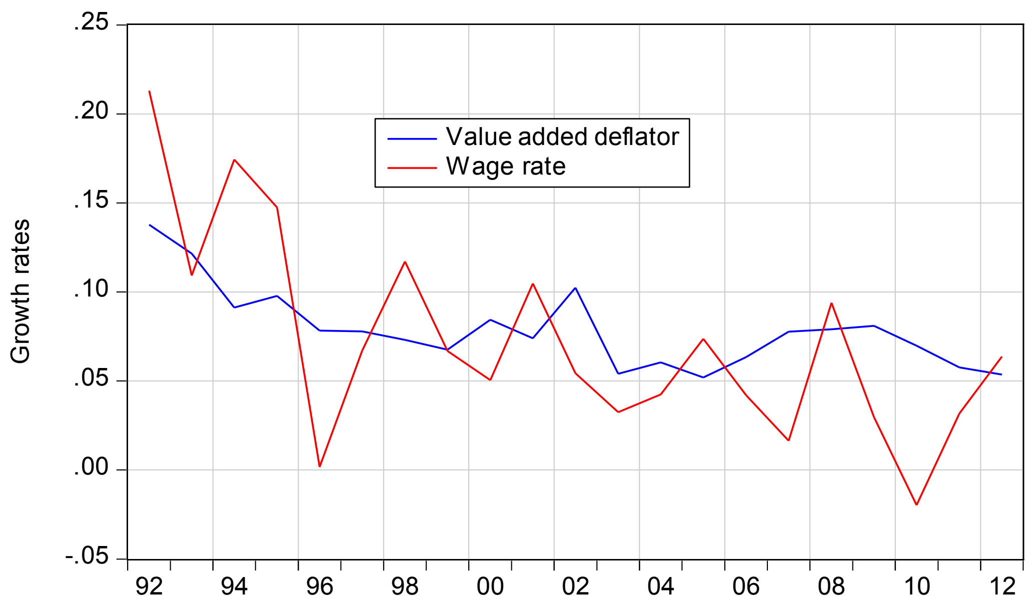
Unfortunately, the estimation gives poor results, both on the role of the rate of use and the speed of correction. To give the model acceptable properties, we will have to calibrate the coefficients.

Dependent Variable: DLOG(PQ)
Method: Least Squares
Date: 04/02/14 Time: 16:34
Sample: 1992 2012
Included observations: 21
DLOG(PQ)=C_PQ(1)*DLOG(WR/LP)+C_PQ(2)*LOG(PQ(-1)/(WR(-1)/LP(
-1)))+C_PQ(5)

	Coefficient	Std. Error	t-Statistic	Prob.
C_PQ(1)	0.235631	0.069667	3.382236	0.0033
C_PQ(2)	-0.042835	0.044422	-0.964262	0.3477
C_PQ(5)	0.081420	0.021534	3.781048	0.0014

R-squared	0.413988	Mean dependent var	0.078780
Adjusted R-squared	0.348875	S.D. dependent var	0.022007
S.E. of regression	0.017758	Akaike info criterion	-5.092418
Sum squared resid	0.005676	Schwarz criterion	-4.943201
Log likelihood	56.47039	Hannan-Quinn criter.	-5.060034
F-statistic	6.358039	Durbin-Watson stat	1.569602
Prob(F-statistic)	0.008150		

This had to be expected, as we can observe no link between the growth rates.



3.4.3.2 The Wage Rate

We suppose that the wage cost is indexed in the long run: for 50% on the value added deflator (the firms want to stabilize the share of wages in value added) and for 50% on the consumption deflator (workers want their purchasing power to follow the gains in productivity). The weighting could be adapted through a parameter.

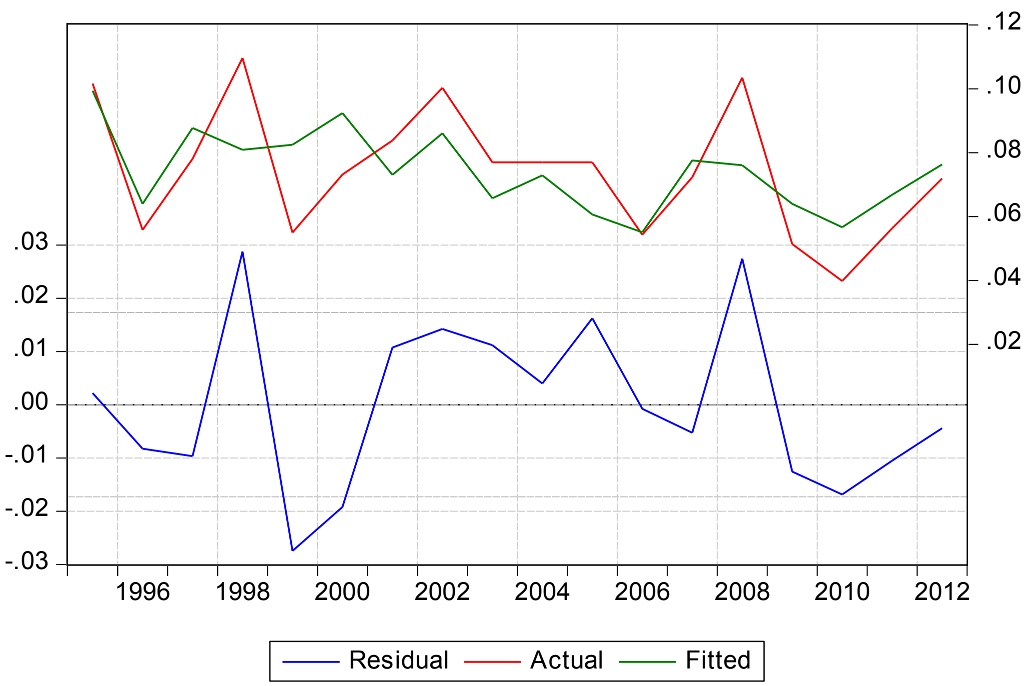
Unfortunately, only the dynamic terms show any significance.

This means we shall have to calibrate the equation if we want our model to present acceptable properties see next page).

Dependent Variable: DLOG(WR)
Method: Least Squares
Date: 04/02/14 Time: 17:48
Sample: 1995 2012
Included observations: 18
DLOG(WR)=C_WR(1)*DLOG(PCOH)+C_WR(2)*LOG(UNR)-C_WR(3)
*(LOG(COSTW(-1))-0.5*LOG(PCOH(-1))-(1-0.5)*LOG(PQ(-1)))
+C_WR(4)

	Coefficient	Std. Error	t-Statistic	Prob.
C_WR(1)	0.613302	0.229266	2.675073	0.0181
C_WR(2)	-0.005297	0.039583	-0.133829	0.8954
C_WR(3)	0.000545	0.058134	0.009373	0.9927
C_WR(4)	0.049249	0.129030	0.381690	0.7084

R-squared	0.384029	Mean dependent var	0.074323
Adjusted R-squared	0.252035	S.D. dependent var	0.020016
S.E. of regression	0.017311	Akaike info criterion	-5.081816
Sum squared resid	0.004195	Schwarz criterion	-4.883955
Log likelihood	49.73634	Hannan-Quinn criter.	-5.054534
F-statistic	2.909450	Durbin-Watson stat	2.149501
Prob(F-statistic)	0.071600		



A cointegration test works, but this does not make the dynamic equation better.

Date: 04/02/14 Time: 17:53
Sample (adjusted): 1996 2012
Included observations: 17 after adjustments
Trend assumption: Linear deterministic trend (restricted)
Series: LOG(WR*LF/Q)-0.5*LOG(PCOH)-0.5*LOG(PQ) LOG(UNR)
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.745499	30.10443	25.87211	0.0140
At most 1	0.331287	6.840802	12.51798	0.3614

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.745499	23.26363	19.38704	0.0130
At most 1	0.331287	6.840802	12.51798	0.3614

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegrating Coefficients (normalized by b'*S11*b=I):

LOG(WR*LF/Q0.5* LOG(PCOH)- 0.5*LOG(PQ)	LOG(UNR)	@TREND(61)
15.88281	6.381811	0.295325
-31.07533	8.616335	-0.565075

Unrestricted Adjustment Coefficients (alpha):

D(LOG(WR*LF/Q)- 0.5*LOG(PCOH)- 0.5*LOG(PQ))	-0.002096	0.017903
D(LOG(UNR))	-0.050947	-0.013662

1 Cointegrating Equation(s):	Log likelihood	69.97097

Normalized cointegrating coefficients (standard error in parentheses)		
LOG(WR*LF/Q)- 0.5*LOG(PCOH)- 0.5*LOG(PQ)	LOG(UNR)	@TREND(61)
1.000000	0.401806	0.018594
	(0.10713)	(0.00258)
Adjustment coefficients (standard error in parentheses)		
D(LOG(WR*LF/Q)- 0.5*LOG(PCOH)- 0.5*LOG(PQ))	-0.033288	
	(0.13713)	
D(LOG(UNR))	-0.809184	
	(0.16771)	

3.4.3.3 The Trade Deflators

For this equation we will apply as usual an error correction format. As explained earlier, exporters take into account their costs, i.e. the production price, which includes wages, the cost of input and margins (which could be excluded to give the actual cost), as well as the price decided by their competitors. All these elements should be measured in the same currency. The formula will use elasticities.

In the long run, a given increase in both local and foreign production prices should have the same impact on the trade prices. A time trend will convey the fact that imports are gradually attracted by the countries with the lowest prices.

Our formula for the export price will be

$$\Delta Log\big(PX(t)\big) = a\,.\Delta Log\big(PP(t)\big) + b.\Delta Log\big(PPX(t).ER(t)\big) + c.(Log(PP(t-1) \\ - d.Log\big(PX(t-1)\big) - (1-d).Log\big(PPX(t-1).ER(t-1)\big) + e.t + f$$

and for the import price

$$\Delta Log\big(PM(t)\big) = a\,.\Delta Log\big(PP(t)\big) + b.\Delta Log\big(PPX(t).ER(t)\big) + c.(Log(PP(t-1) \\ - d.Log\big(PM(t-1)\big) - (1-d).Log\big(PPX(t-1).ER(t-1)\big) + e.t + f$$

We have restricted the estimation period to the last twenty years, for economic and statistical reasons.

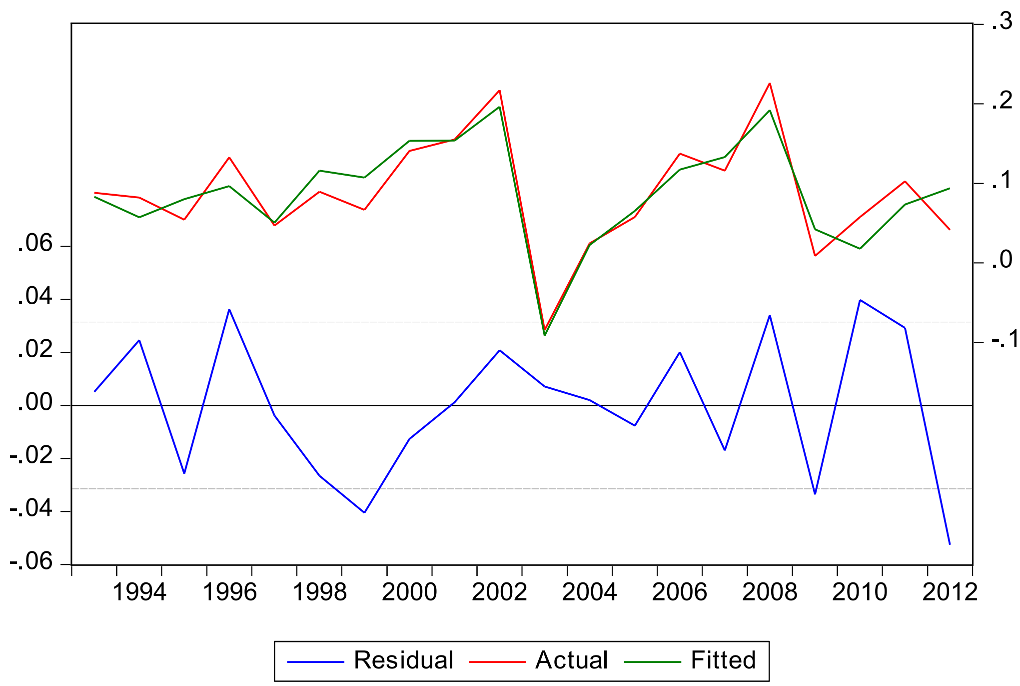
The estimations are surprisingly successful, both in economic and statistical terms.

[26] This could be formalized easily, but we hope the message is already clear.

Dependent Variable: DLOG(PX)
Method: Least Squares
Date: 04/02/14 Time: 18:34
Sample: 1993 2012
Included observations: 20
Convergence achieved after 5 iterations
DLOG(PX)=C_PX(1)*DLOG(PP)+C_PX(2)*DLOG(PPX*ER)-C_PX(3)
*(LOG(PX(-1))-C_PX(4)*LOG(PP(-1))-(1-C_PX(4))*LOG(PPX(-1)*ER(
-1)))+C_PX(5)*(T-2012)*(T<=2012)+C_PX(6)

	Coefficient	Std. Error	t-Statistic	Prob.
C_PX(1)	1.481106	0.814188	1.819120	0.0903
C_PX(2)	0.422891	0.105830	3.995944	0.0013
C_PX(3)	0.424417	0.165671	2.561814	0.0226
C_PX(4)	0.717053	0.179198	4.001464	0.0013
C_PX(5)	0.012435	0.004274	2.909039	0.0114
C_PX(6)	0.061050	0.035072	1.740709	0.1037

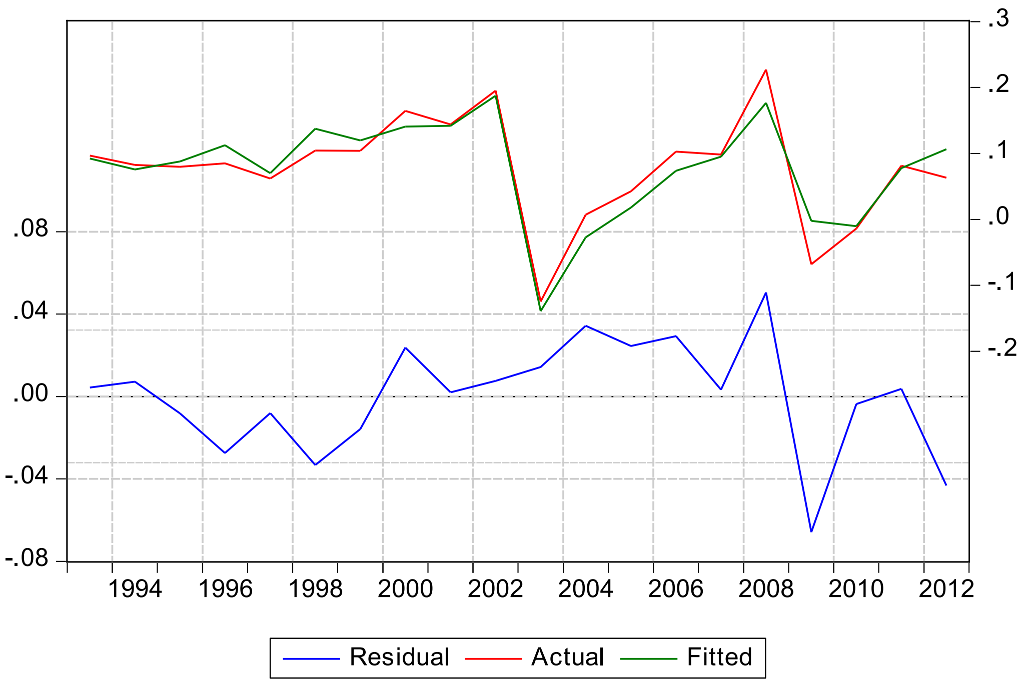
R-squared	0.855231	Mean dependent var	0.088095
Adjusted R-squared	0.803528	S.D. dependent var	0.070987
S.E. of regression	0.031465	Akaike info criterion	-3.836540
Sum squared resid	0.013861	Schwarz criterion	-3.537820
Log likelihood	44.36540	Hannan-Quinn criter.	-3.778226
F-statistic	16.54121	Durbin-Watson stat	2.327903
Prob(F-statistic)	0.000019		



Dependent Variable: DLOG(PM)
Method: Least Squares
Date: 04/02/14 Time: 18:37
Sample: 1993 2012
Included observations: 20
Convergence achieved after 1 iteration
DLOG(PM)=C_PM(1)*DLOG(PP)+C_PM(2)*DLOG(PPX*ER)-C_PM(3)
*(LOG(PM(-1))-C_PM(4)*LOG(PP(-1))-(1-C_PM(4))*LOG(PPX(-1)*ER(
-1)))+C_PM(5)*(T-2012)*(T<=2012)+C_PM(6)

	Coefficient	**Std. Error**	**t-Statistic**	**Prob.**
C_PM(1)	0.982132	0.721113	1.361966	0.1947
C_PM(2)	0.587004	0.091315	6.428339	0.0000
C_PM(3)	0.533116	0.209915	2.539676	0.0236
C_PM(4)	0.426463	0.134416	3.172717	0.0068
C_PM(5)	0.005477	0.002647	2.069170	0.0575
C_PM(6)	-0.011645	0.037053	-0.314266	0.7580

R-squared	0.886419	Mean dependent var	0.076752
Adjusted R-squared	0.845854	S.D. dependent var	0.082109
S.E. of regression	0.032237	Akaike info criterion	-3.788062
Sum squared resid	0.014549	Schwarz criterion	-3.489343
Log likelihood	43.88062	Hannan-Quinn criter.	-3.729749
F-statistic	21.85201	Durbin-Watson stat	1.861828
Prob(F-statistic)	0.000004		



As usual for this type of equation, exporters show a higher attention to their costs than to the price of their competitors.

This will dampen the dynamics of the price-wage loop. The impact of trade on the price of demand can be explained in the following way.

- Imports are a share of global demand. They are bought at the import price. The higher its sensitivity to foreign costs, the higher the difference to the local production price, and the higher the reducing impact of imports on the global demand price.
- If local producers decided on their selling price on the local and foreign markets independently (a possible behavior that we did not consider), the sensitivity of the demand price to local costs would clearly be less than one.
- But in our framework, the production price is decided globally, and the lower sensitivity of the export price has to be balanced by a higher sensitivity of the price at which they sell on the local market. The higher the impact of local costs on the export price, the lower the necessary compensation.

One can see that in the transition from production to demand price, the higher the role of the production cost in the price set by the exporter, the higher the first (negative) effect and the lower the second (positive) one.[27]

In the extreme, if all exporters take only into account their costs, the import price will not be affected, and as the export price will change just as the global production price, no compensation will be needed. The damping effect will be maximal; contrary to the general case, however, the additional trend is positive (and significant).

To reach a steady state in the long run, these trends will have to be suppressed after a while. Here we did it immediately, but true forecasts should call for a gradual decrease.

3.4.4 Household consumption

Our equation follows as usual an error correction specification (estimated in one step!) following almost completely the framework presented earlier.

The change in consumption depends on:

- The change in real income (over the last year).
- The change in unemployment.
- The short term real interest rate
- An error correction term.

[27] This could be formalized easily, but we hope the message is already clear.

- A negative time trend, representing the increase in the wealth of households and their saving potential (in particular their accession to housing ownership).
- The past change in consumption, representing the inertia of past habits.

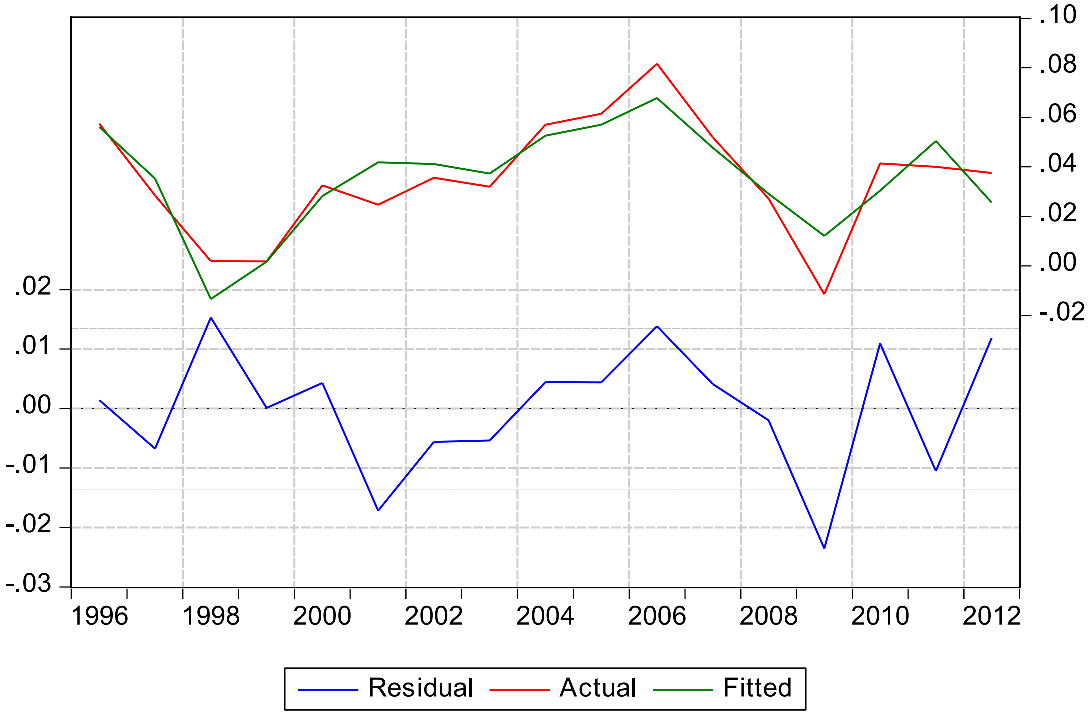
The only influence we could not evidence is that of inflation (for which the sign is uncertain, anyway).

The fit is quite good, with maybe too many explanations for such a small sample. Also, the coefficient for the change in revenue is quite high (almost unitary).

Dependent Variable: DLOG(COH)
Method: Least Squares
Date: 04/03/14 Time: 09:11
Sample: 1996 2012
Included observations: 17
DLOG(COH)=C_COH(1)+C_COH(2)*(T-2012)*(T<=2012)+C_COH(3)
*DLOG(HRDI)+C_COH(4)*LOG(COH(-1)/HRDI(-1))+C_COH(5)*D(UNR)
+C_COH(6)*(IRS-100*@PCH(PCOH))+C_COH(7)*DLOG(COH(-1))
+COH_EC

	Coefficient	Std. Error	t-Statistic	Prob.
C_COH(1)	-0.271889	0.086498	-3.143281	0.0105
C_COH(2)	-0.004488	0.001295	-3.466474	0.0061
C_COH(3)	0.976120	0.209552	4.658127	0.0009
C_COH(4)	-0.591870	0.189123	-3.129550	0.0107
C_COH(5)	-0.003871	0.003076	-1.258432	0.2368
C_COH(6)	-0.003820	0.001747	-2.185839	0.0537
C_COH(7)	0.488962	0.171620	2.849097	0.0173

R-squared	0.791731	Mean dependent var	0.035383
Adjusted R-squared	0.666770	S.D. dependent var	0.023440
S.E. of regression	0.013531	Akaike info criterion	-5.474764
Sum squared resid	0.001831	Schwarz criterion	-5.131676
Log likelihood	53.53549	Hannan-Quinn criter.	-5.440660
F-statistic	6.335808	Durbin-Watson stat	2.363382
Prob(F-statistic)	0.005627		



3.4.5 Exports

For exports we shall use again an error correction framework, estimated in one pass.

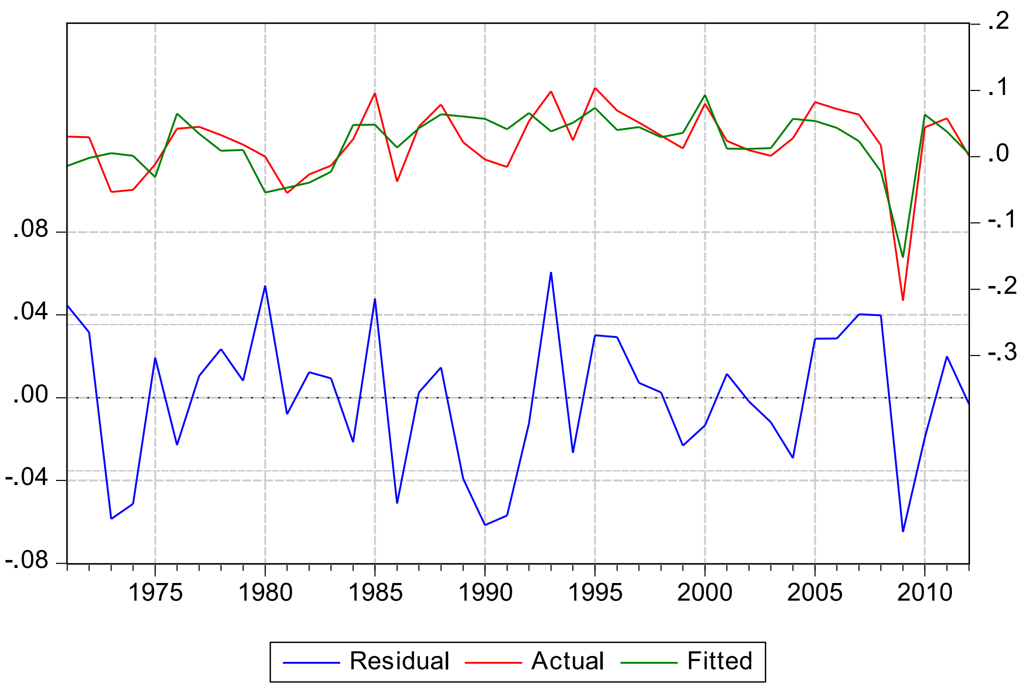
Unfortunately we were not able to evidence a significant role of the rate of use, and the coefficient of price competitiveness is quite low. Also, a significant trend had to be introduced.

Finally, the estimation gives better results on a large period (due obviously to the number of observations). It would mean that the exports behavior has not changed since the disappearance of apartheid.

Dependent Variable: DLOG(X)
Method: Least Squares
Date: 04/03/14 Time: 09:33
Sample (adjusted): 1971 2012
Included observations: 42 after adjustments
DLOG(X)=C_X(1)*DLOG(WD)+C_X(2)*LOG(X(-1)/WD(-1))+C_X(3)
LOG(COMPX)+C_X(4)+C_X(5)(T-2012)*(T<=2012)

	Coefficient	Std. Error	t-Statistic	Prob.
C_X(1)	0.815319	0.124863	6.529705	0.0000
C_X(2)	-0.516027	0.110380	-4.674999	0.0000
C_X(3)	-0.113171	0.032689	-3.462109	0.0014
C_X(4)	-1.855945	0.395269	-4.695392	0.0000
C_X(5)	-0.013248	0.002943	-4.501494	0.0001

R-squared	0.635408	Mean dependent var	0.021601
Adjusted R-squared	0.595993	S.D. dependent var	0.055509
S.E. of regression	0.035282	Akaike info criterion	-3.739516
Sum squared resid	0.046060	Schwarz criterion	-3.532651
Log likelihood	83.52984	Hannan-Quinn criter.	-3.663692
F-statistic	16.12086	Durbin-Watson stat	1.827690
Prob(F-statistic)	0.000000		



Imports

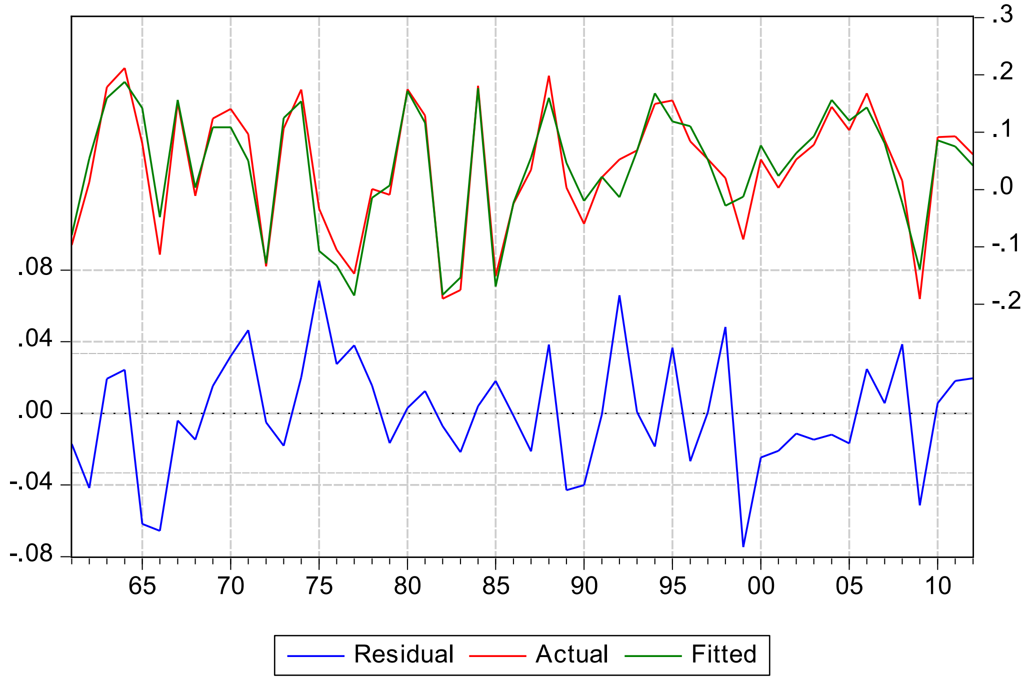
For imports we use the same framework, but again, we could not evidence an influence of the rate of use. The short term coefficient of demand is very high, probably to compensate this loss. The coefficient of price competitiveness is quite low. We had to introduce a positive time trend. As for exports, we are considering a very large period.

The quality of the fit is really surprising.

Dependent Variable: DLOG(M)
Method: Least Squares
Date: 04/03/14 Time: 10:55
Sample (adjusted): 1961 2012
Included observations: 52 after adjustments
DLOG(M)=C_M(1)*DLOG(TD)+C_M(2)*LOG(COMPM)+C_M(3)+C_M(4)
LOG(M(-1)/TD(-1))+C_M(5)(T-2012)*(T<=2012)

	Coefficient	Std. Error	t-Statistic	Prob.
C_M(1)	3.051122	0.152208	20.04578	0.0000
C_M(2)	-0.218070	0.036083	-6.043558	0.0000
C_M(3)	-0.584799	0.060871	-9.607188	0.0000
C_M(4)	-0.276709	0.028813	-9.603534	0.0000
C_M(5)	0.002497	0.000348	7.186536	0.0000

R-squared	0.914777	Mean dependent var	0.038646
Adjusted R-squared	0.907524	S.D. dependent var	0.109453
S.E. of regression	0.033285	Akaike info criterion	-3.876232
Sum squared resid	0.052070	Schwarz criterion	-3.688613
Log likelihood	105.7820	Hannan-Quinn criter.	-3.804303
F-statistic	126.1230	Durbin-Watson stat	1.805598
Prob(F-statistic)	0.000000		



3.4.6 Conclusion

On the whole, this version of the model is rather consistent both in terms of statistics and economics. The only elements which are not acceptable are the estimation of the wage rate and the value added deflator. As the other price equations (the trade prices) look satisfying, it is quite possible that the problems come from the wage data, which enters in both. Relying on official series from the local statistical institute (which we do not find on its website) could be a solution to the problem. This applies also to labor, even though the capacity and labor equations look acceptable.

The absence of a significant impact of available supply in the trade equations is another issue. The associated mechanism is necessary if we want the model properties to be realistic. It mean we will have to resort to calibration. For the same reason, we will also have to calibrate the role of unemployment in wages and of supply in the value added deflator. But using another source for the wage rate might solve the problem.

3.5 The Fourth Task: Simulating the Model

Now that we have established the model, we can simulate it. But first, we have to check that the data we obtained earlier, and the equations we have established, are consistent with each other. For this, we shall use a technique called "Residual check".

3.5.1 A First Test: Checking the Residuals In The Identities

This method will compute each formula in the model using the historical values of the variables. This can be done by creating for each equation a formula giving the value of the right-hand side expression (using the GENR statement in EViews); however, EViews provides a much simpler method.

If we consider a model written as:

$$y_t = f(y_t, y_{t-1}, x_t, \hat{\alpha})$$

with y and x the vectors of endogenous and exogenous variables, we can perform a very specific "simulation", in which each equation is computed separately using historical values.

Technically, this means:

- Breaking down the model into single equation models, as many as there are equations.
- Solving each of these models separately, using as explanatory values the historical ones. If we call these historical values y_t^0

It means we shall compute:

$$y_t = f(y_t^0, y_{t-1}^0, x_t, \hat{\alpha}) + e_t$$

One has two consider two cases:

- For identities, the computation should give the historical result. Otherwise there is at least one error[28].
- For the behavioral equations, if we have introduced the estimated residual as an additional term, computing the estimated equation will give the historical result.

EViews actually allows the use of an expression on the left hand side. This applies also here, the comparison being made between the left and right expressions.

The interest of this method is obvious: if the residual in the equation is not zero, it means that there is *at least* one error in that *particular* equation. Of course the problem is not solved, but its location is identified.

It would be illusory, however, to hope to obtain a correct model immediately: some error diagnoses might have been badly interpreted, and corrections badly performed. But even if the error has been corrected in the right way:

[28] A zero residual does not guarantee the absence of error, as two errors might compensate each other, for instance if the model uses the formula which computed the series.

- There could be several errors in the same equation
- The correcting process can introduce an error in another equation that looked previously exact, but contained actually two balancing errors.

Of course, in our case, all the residuals are now zero (a result which took some time to obtain).

3.5.2 Simulating the Model over the Past

Now the model is ready to be simulated (actually if all residuals are zero, any simulation should converge to the historical values).

To solve the model we need to apply a method. Let us present the different algorithms.

3.5.2.1 Gauss-Seidel

This is the most natural algorithm. Let us formalize this process.

Considering the model

$$y_t = f_t(y_t, y_{t-1}, x_t, \hat{\alpha})$$

in which we will only consider present elements

$$y = f(y)$$

we will use an exponent to define the iteration count.

a - We start from y^0, value at iteration 0.

b - We add 1 to the number of iterations (which we shall note k).

c -We compute y_i^k from i = 1 to n, taking into account the i-1 values we have just produced. This means we compute:

$$y_i^k = f(y_1^k, \ldots, y_{i-1}^k, y_i^{k-1}, \ldots., y_n^{k-1})$$

d – We compare y^k and y^{k-1}: if the distance is small enough for every element we stop the process, and take as solution the last value. If not, we repeat until the condition is met (or a maximum number of iterations is reached).

Clearly, this algorithm requests an identified model (with y on the left).

3.5.2.2 Newton

Contrary to Gauss Seidel, the Newton method applies naturally to non-identified formulations. It represents actually a generalization to an n-dimensional problem of the well-known method using a sequence of linearizations to solve a single equation.

Let us consider the model

$$f_t(y_t, y_{t-1}, x_t, \hat{\alpha}) = 0$$

that we will simplify as above into

$$f\ (y) = 0$$

The linearization of f around a starting solution gives, by calling "fl" the value of f linearized,

$$(\partial f / \partial y)_{y=y^0} \cdot (y - y^0) = fl(y) - f(y^0)$$

Solving the system for fl (y) = 0 leads to:

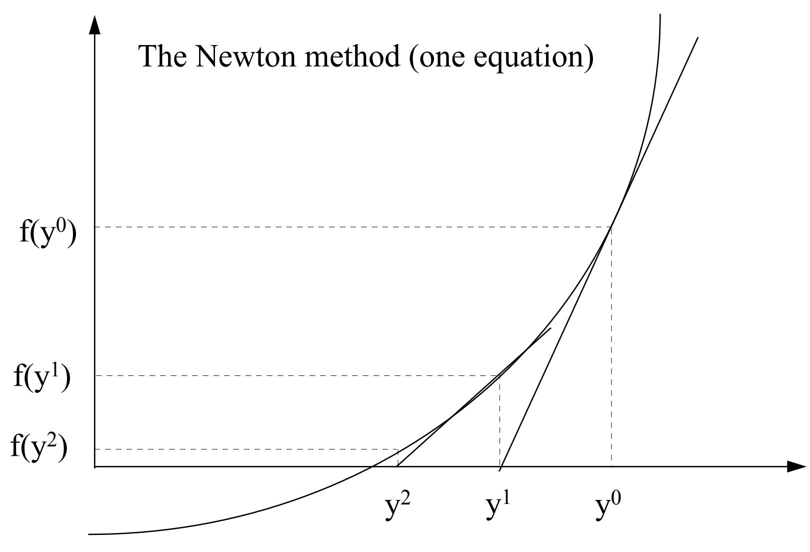
$$y = y^0 - (\partial f / \partial y)^{-1}_{y=y^0} \cdot f(y^0)$$

With an identified system

$$y - f\ (y) = 0$$

we would get:

$$y = y^0 - (I - \partial f / \partial y)^{-1}_{y=y^0} \cdot (y^0 - f(y^0))$$



3.5.2.3 Broyden's method

Broyden's method (also called secant method) computes the Jacobian only once, similar to Newton's method, and computes a new value of the variable accordingly.

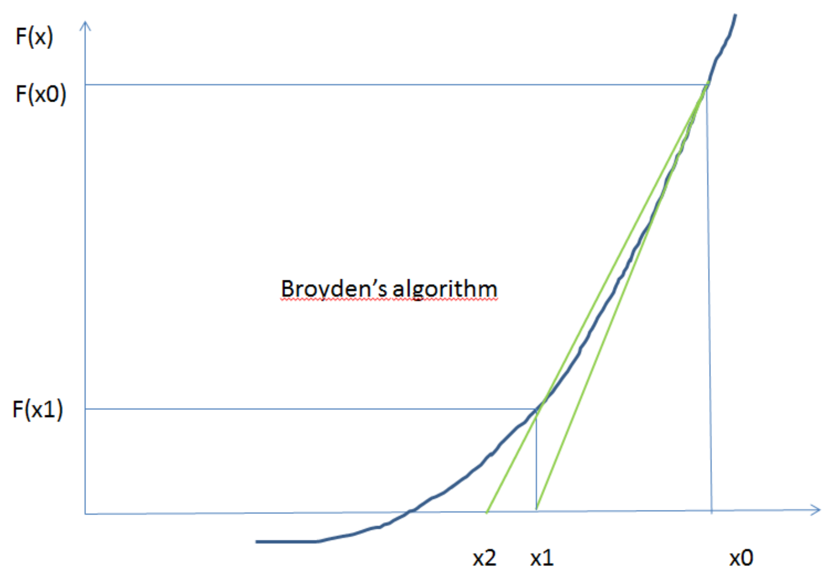
After that, it updates the Jacobian; not by derivation, but by considering the difference between the previous one and the direction leading from the previous solution to the new one.

The formula for updating the Jacobian is:

$$J_{t+1}=J_t + (F(x_{t+1}) - F(x_t) - J_t\Delta x_t).\Delta x'_t/(\Delta x'_t.\Delta x_t)$$

where J is the Jacobian, F the function which should reach zero, and x the vector of unknown variables.

Let us clarify all this with a graph based on the single equation case.



We can see that the direction improves with each iteration, less than Newton but more than Gauss-Seidel (for which it does not improve at all).

Otherwise the method shares all the characteristics of Newton's, in particular its independence on equation ordering. It takes generally more iterations, but each of them is cheaper (except for the first).

We shall see that on average it looks like the most efficient option on the whole, both in terms of speed and probability of convergence[29]. But the diagnosis is not so clear cut.

3.5.3 Testing the Model over the Future

The tests performed on the sample period were not so satisfying, for several reasons:

[29] The most important feature in our opinion.

- For simulations, even if it was the only way to check results against actual data, the fact that this data had actually been used to produce the estimations could not avoid the tests from being flawed, whatever the precautions we had taken.
- For shock analysis, the sample period was generally too short to evidence long term properties and to measure cycles, and the irregularities (to say the least) in the base trajectory have been transmitted to the results (due to the non-linearity of the model), making the diagnosis on model stability unclear.
- Moreover, the results applied to the historical period, which is not the true field for future operational uses of the model.
- To obtain enough information, shocks have to be conducted on at least ten years, making the starting period quite far from the present.

This leads to the natural idea: test the model on the future.

- We shall have initial information on the reliability of spontaneous forecasts, and of their distance to what we expect of the near future.
- The results will me more representative of future use.
- The actual results can be interpreted as the actual consequences of policy decisions.
- The first periods of the shock will be representative of the present efficiency of present policies.
- The tests can be conducted on a period of any length, allowing to observe convergence and cycles.
- With regularly growing assumptions we can test that the simulation is regular, and that it converges to both a steady state growth and a long term stable solution.
- Applying to these regular solution constant shocks we can check that we get smooth evolutions, and we can interpret them easier.
- We have enough observations to treat the Lucas critique.

There are only two drawbacks:

- We cannot check the simulation results against true values. We shall try to prove this is not so important, and can be replaced by other tests.
- We do have to produce a simulation over the future, an unknown domain in which convergence might be more difficult to achieve.

3.5.4 The Results

We do not know enough about South African economy and its current prospects to be able to produce a reliable forecast. Our goal will be only to assess if, under reasonable assumptions on the foreign environment and Government policy, the model provides reasonable economic evolutions.

For this reason, we will speak of "simulations over the future" rather than actual "forecasts."

3.5.4.1 A Very First Simulation

We will simulate the model over a rather long period, actually 2013-2050. The reason for this, as we have already stated, is to control its convergence to a solution, and to interpret visually its dynamics, in particular the presence of potential cycles.

The only element we have to define is the value of assumptions.

Here is a list of model assumption, ordered by dimension. We can observe that a large majority of them have no dimension, which means that their evolution is independent from the economic context. If we defined

- the potential work force as a share of total population,
- government employment as a share of the potential work force,
- remittances as a share of world demand, and
- government investment and GDP as a share of GDP,

the only dimensioned assumptions would be world demand and prices, which means that the country's economy would converge to a given share of the world GDP and a given ratio to world prices.

name	units	rate	definition
COH_EC	ratio	0	Residual on household consumption
ERX	deflator base 2005	0	Exchange rate (exogenous)
FDXR	ratio	0	Residual demand
IC_EC	ratio	0	Residual on change in inventories
IP_EC	ratio	0	Residual on Productive investment
IR_EC	ratio	0	Residual on the interset rate
IRL_EC	ratio	0	Residual on the long term interest rate
LF_EC	ratio	0	Residual on firms employment
M_EC	ratio	0	Residual on imports
NIF_ER	ratio	0	Residual on firms net interests paid
NIG_ER	ratio	0	Interests paid by Government
NIXD_EC	ratio	0	Residual on interests paid to the RoW in foreign currency
NIXX_EC	ratio	0	Residual on interests paid to the RoW in local currency
PM_EC	ratio	0	Residual on the imports deflator

POPAC_EC	ratio	0	Residual on the work force
PQ_EC	ratio	0	Residual on the value added deflator
PX_EC	ratio	0	Residual on the exports deflator
R_EXPG	ratio	0	Residual on Government expenditures
R_ICT	ratio	0	Income tax rate
R_IFP	ratio	0	Rate of the tax on Firms profits
R_IH	ratio	0	Ratio of housing investment to revenue
R_LW	ratio	0	Share of wage earners in firms employment
R_OIT	ratio	0	Other indirect taxes rate
R_PCOG	ratio	0	Ratio of the household consumption deflator to the global demand price
R_PCOH	ratio	0	Ratio of the government consumption deflator to the global demand price
R_PIG	ratio	0	Ratio of the Government investment price to the global demand deflator
R_PIP	ratio	0	Ratio of the firms investment price to the global demand deflator
R_REVG	ratio	0	Residual on Government revenue
R_REVQ	ratio	0	Other household revenue based on GDP
R_REVX	ratio	0	Other household revenue not based on GDP
R_SCF	ratio	0	Rate of social security contributions paid by firms
R_SCG	ratio	0	Rate of social security contributions paid by Government
R_SCW	ratio	0	Rate of social security contributions paid by households
R_SUBS	ratio	0	Rate of firms subsidies to Vue added
R_TAR	ratio	0	Rate of local tariffs
R_TARX	ratio	0	Rate of foreign tariffs
R_VAT	ratio	0	VAT rate
RDEP	ratio	0	Depreciation rate of capital
RES_FDV	ratio	0	Residual on final demand at current prices
TC	ratio	0	Technical coefficient
URD	ratio	0	Target capacity utilization rate
WR_EC	ratio	0	Residual on the wage rate
X_EC	ratio	0	Residual on exports
IRMX	points	0	Interest rate, average on current debt, exogenous
IRSR	points	0	Interest rate, short term, real
IRST	points	0	Interest rate, Taylor residual
IRSX	points	0	Interest rate, short term exogenous
IRX	points	0	Interest rate, foreign
COG	constant 2005 rands	txq	Government consumption (real)
IG	constant 2005 rands	txq	Government investment
REMX	constant 2005 rands	txq	Remittances
SOCBR	constant 2005 rands	txq-txn	Social benefits
WD	constant 2005 US dollars	txq	World demand
PPX	deflator base 2005	txp	Deflator of foreign production
LG	persons	txq	Employment of Government

POP1564	persons	txn	Population in age of working
POPT	persons	txn	Population, total

Using the following rates, consistent with the recent evolution of South African economy:

txq=0.035 (quantities)

txn=0.01 (populations)

txp=0.050 (prices)

Results are presented on the next page in Table 2.

A table which shows very regular evolutions, with a growth rate pf GDP and employment close to the theoretical values, but an inflation significantly lower, at least in the short-medium run.

It would not be too difficult to improve the results, and also to introduce an increase of world trade. For instance, increasing the residual on the value added deflator by 1 point, we get the results are presented in Table 3.

Table 2. South Africa: A very first simulation over the future

	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2025
Real equilibrium											
GDP	3.59	3.49	3.55	3.63	3.70	3.77	3.81	3.84	3.86	3.87	3.85
Final demand	4.92	4.11	3.83	3.72	3.69	3.69	3.69	3.70	3.71	3.71	3.71
Productive Investment	4.69	4.33	4.29	4.30	4.31	4.31	4.30	4.27	4.23	4.19	4.03
Total Investment	4.23	4.01	3.98	3.99	4.00	4.01	4.00	3.98	3.96	3.93	3.83
Household Consumption	5.26	4.30	3.86	3.68	3.62	3.62	3.64	3.66	3.69	3.71	3.73
Imports	7.57	5.59	4.67	4.14	3.82	3.62	3.50	3.42	3.37	3.34	3.30
Exports	2.70	3.48	3.80	3.92	3.95	3.95	3.92	3.90	3.87	3.84	3.77
Export-import ratio	70.66	69.24	68.67	68.52	68.60	68.81	69.10	69.42	69.76	70.10	71.08
Productive capacity	3.27	3.28	3.33	3.41	3.49	3.56	3.63	3.68	3.72	3.76	3.80
Rate of use (*)	113.13	113.34	113.57	113.81	114.05	114.27	114.48	114.66	114.81	114.94	115.17
Deflators											
GDP	3.43	3.42	3.44	3.49	3.56	3.64	3.73	3.81	3.90	3.99	4.24
Exports	1.44	2.63	3.24	3.58	3.78	3.91	4.01	4.09	4.16	4.23	4.41
Imports	2.47	3.58	4.15	4.44	4.59	4.68	4.73	4.77	4.79	4.81	4.86
Consumer price index	3.79	3.71	3.71	3.76	3.82	3.89	3.97	4.04	4.11	4.18	4.38
Production	3.60	3.56	3.57	3.62	3.69	3.76	3.84	3.92	4.00	4.08	4.31
Wage rate	5.38	5.58	5.75	5.91	6.06	6.20	6.33	6.46	6.57	6.68	6.96
Export competitiveness	-3.39	-2.26	-1.67	-1.35	-1.17	-1.04	-0.95	-0.87	-0.80	-0.73	-0.56
Import competitiveness	-1.09	0.02	0.56	0.79	0.87	0.88	0.86	0.81	0.76	0.70	0.53
Employment											
Firms employment	1.13	1.02	1.03	1.09	1.17	1.23	1.28	1.32	1.34	1.36	1.35
Total employment	1.13	1.02	1.03	1.09	1.17	1.23	1.28	1.32	1.34	1.36	1.35
Unemployment rate (*)	26.10	25.81	25.62	25.47	25.34	25.23	25.12	25.02	24.92	24.83	24.58

Table 3. South Africa: An improved simulation

	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2025
Real equlibrium											
GDP	3.30	3.13	3.16	3.23	3.32	3.41	3.48	3.55	3.60	3.65	3.72
Final demand	4.58	3.78	3.50	3.41	3.40	3.42	3.46	3.49	3.53	3.55	3.61
Productive Investment	4.20	3.74	3.66	3.69	3.76	3.83	3.89	3.93	3.96	3.98	3.96
Total Investment	3.93	3.65	3.60	3.62	3.66	3.70	3.74	3.77	3.79	3.80	3.79
Household Consumption	4.90	3.92	3.47	3.30	3.26	3.28	3.32	3.38	3.43	3.48	3.58
Imports	7.21	5.41	4.59	4.13	3.85	3.68	3.57	3.50	3.45	3.42	3.36
Exports	2.55	3.24	3.53	3.66	3.71	3.73	3.74	3.75	3.75	3.75	3.73
Export-import ratio	70.78	69.33	68.63	68.32	68.22	68.26	68.37	68.53	68.73	68.95	69.69
Productive capacity	3.17	3.10	3.10	3.14	3.21	3.27	3.34	3.41	3.46	3.51	3.62
Rate of use (*)	112.93	112.95	113.01	113.10	113.22	113.37	113.52	113.68	113.84	113.99	114.37
Deflators											
GDP	4.99	4.69	4.48	4.34	4.24	4.18	4.15	4.14	4.14	4.16	4.26
Exports	2.29	3.46	4.00	4.22	4.31	4.34	4.35	4.36	4.37	4.38	4.45
Imports	2.76	3.81	4.34	4.59	4.72	4.78	4.81	4.82	4.83	4.84	4.86
Consumer price index	5.13	4.76	4.55	4.43	4.36	4.32	4.29	4.29	4.29	4.31	4.40
Production	5.06	4.73	4.52	4.38	4.30	4.25	4.22	4.21	4.21	4.23	4.32
Wage rate	6.21	6.22	6.25	6.29	6.33	6.38	6.44	6.50	6.57	6.63	6.84
Export competitiveness	-2.58	-1.46	-0.96	-0.74	-0.65	-0.62	-0.62	-0.61	-0.60	-0.59	-0.53
Import competitiveness	-2.19	-0.87	-0.17	0.20	0.40	0.51	0.57	0.59	0.59	0.59	0.52
Employment											
Firms employment	0.94	0.72	0.67	0.71	0.79	0.87	0.95	1.02	1.08	1.12	1.21
Total employment	0.94	0.72	0.67	0.71	0.79	0.87	0.95	1.02	1.08	1.12	1.21
Unemployment rate (*)	26.15	25.95	25.85	25.80	25.76	25.73	25.69	25.65	25.61	25.56	25.40

3.5.4.2 Producing Shocks on Assumptions

Let us now see how our model answers to shocks on its assumptions. We shall be brief, and only present and comment the results for seven of them: our goal is mostly to show that the present framework and the associated estimations can provide a model with consistent properties. The model(s) we are presenting are far from perfect, and we did not try to make them so.[30]

All shocks will start in 2014, and will be sustained for the whole period. To make interpretation easier, we shall limit the periods to the first twelve years.

Even though the estimation of most formulas has been successful, producing a model with reasonable properties has called for some changes, at which we have generally hinted already.

We have both changed some values, and introduced new mechanisms we could not justify by estimation.

Of course, these values are quite arbitrary (although acceptable from a theoretical point of view) and the reader will be able to test other figures, using the programs which will be described and provided in the second part.

Their choice comes both from our experience in successful cases, and the observation of the properties they give to the model.

When coefficients have been added or changed, we have not re-estimated the equations, except for the constant term.

3.5.4.2.1 The value added deflator

We have added an impact of the rate of use, with a 0.30 coefficient. As we have just said, this value comes prom experience and tests on model properties.

We have also reduced the dynamic impact of the wage cost to 0.75, and increase the speed of convergence to 0.20.

The equation becomes.

*DLOG(PQ) = 0.480*DLOG(COSTW) + 0.3*DLOG(UR) - 0.0332*(LOG(PQ(- 1)/COSTW(- 1)) - 0.3*LOG(UR(- 1))) + 0.0498 + PQ_EC*

3.5.4.2.2 The wage rate

We have introduced an impact of unemployment, and increased a little the speed of convergence.

*DLOG(WR) = 0.633 *DLOG(PCOH) - 0.15*LOG(UNR) - 0.1*(LOG(COSTW(- 1)) - 0.5*LOG(PCOH(- 1)) - (1 - 0.5)*LOG(PQ(- 1))) + 0.468 + WR_EC*

[30] Actually, we can identify a few coefficients which, with different values, would improve slightly model properties. Of course, we will not apply these changes.

3.5.4.2.3 The imports

We have introduced an influence of the rate of use, substituting partly for that of demand itself, reducing its coefficient to 1.5.

*DLOG(M) = 1.5*DLOG(TD) - 0.0943 *LOG(COMPM) + 1*LOG(UR) + 0.465 - 0.00335 *LOG(M(- 1)/TD(- 1)) + 0.001618 *(T - 2012)*(T <= 2012) + M_EC*

3.5.4.2.4 The exports

We have introduced an influence of the rate of use.

*DLOG(X) = 0.815 *DLOG(WD) - 0.516 *LOG(X(- 1)/WD(- 1)) - 0.3*LOG(UR) - 0.113 *LOG(COMPX) - 1.9761738399233 - 0.01324 *(T - 2012)*(T <= 2012)*

3.5.4.2.5 The trade prices

Even though the estimations were rather acceptable, we have changed most elements, reducing the short term impacts and increasing the long term sensitivity of the export deflator to the local cost.

This is consistent with usual estimations and theory, but we might revert to estimated values in the future, at least partially.

*DLOG(PX) = 0.6*DLOG(PP) + 0.4*DLOG(PPX*ER) - 0.5*(LOG(PX(- 1)) - 0.8*LOG(PP(- 1)) - (1 - 0.8)*LOG(PPX(- 1)*ER(- 1))) + 0.000164*(T - 2012)*(T <= 2012) + 0.0436 + PX_EC*

*DLOG(PM) = 0.2*DLOG(PP) + 0.8*DLOG(PPX*ER) - 0.5*(LOG(PM(- 1)) - 0.2*LOG(PP(- 1)) - (1 - 0.2)*LOG(PPX(- 1)*ER(- 1))) - 0.000572*(T - 2012)*(T <= 2012) + 0.0123 + PM_EC*

3.5.4.3 The Shocks

3.5.4.3.1 An increase in government demand

In this shock, we increase Government demand by 1% of the baseline GDP. This is the first shock one has in mind, as it defines the Keynesian multiplier, showing the way the country's economic mechanisms modify the consequences of an external shock on demand. Technically the multiplier will be the ratio between the ex post and ex-ante changes in GDP, the latter being the change in one of the exogenous demand elements.

Observing the role of Government demand (investment IG or consumption CG) we can see that it affects only:

- Final demand, and GDP through the supply – demand equilibrium.
- Government expenditures and deficit.

As stated earlier, we do not consider the increase in global productivity coming from Government investment.

Let us concentrate on the supply – demand equilibrium:

$$GDP + M = FD + X$$

With:

$$FD = COH + cog + IP + IH + ig + IC$$

In terms of GDP, the ex-ante impact of the shock is of course 1% (this is the reason for the size we have chosen). The evolution to the ex post value comes from the endogenous elements:

- Trade elements: exports and imports.
- Demand elements: consumption, investment, changes in inventories.

Obviously we shall get, for ex post GDP:

- A positive effect from demand, as the need for additional capacities will increase investment, and the new jobs will produce wages, household revenue and consumption.
- But a negative one from imports, as a share of this additional demand will have to be imported.

Two additional effects have to be considered:

- In the short and medium runs, demand will meet a local capacity constraint for some products, which will have to be imported. And as local producers will use a larger part of their capacity to satisfy local demand, they will be less active in looking for export markets.

This effect will disappear gradually as local producers adapt their capacities (through investment).

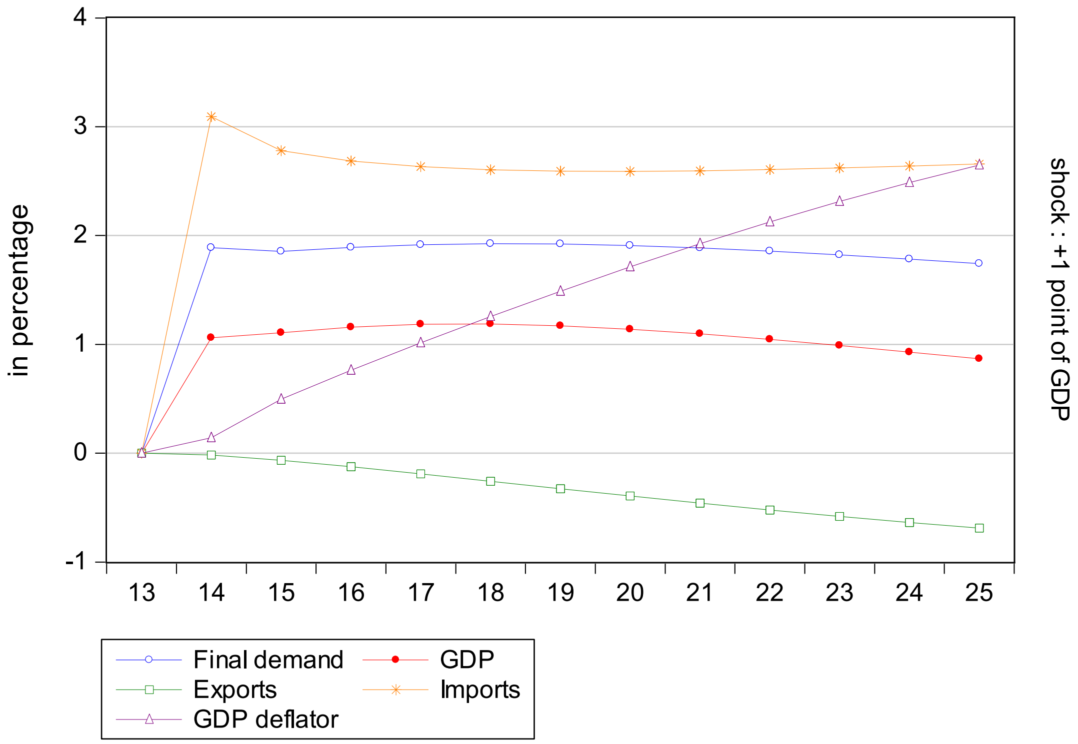
- Inflation will appear, due to disequilibria:
  - On capacities, as the higher level of production (compared to capacities through the rate of use) will allow optimizing firms to increase their prices. This effect will disappear in the long run.
  - On wages, as the lower level of unemployment will improve the negotiating power of workers. As long as unemployment is reduced, this effect will remain.

Inflation will reduce the competitiveness of local producers, both on the foreign and the local markets.

The following graph illustrates our comments. Ex-post demand grows by 1.9% (compared to 1% ex ante, the South African net trade balance being roughly at equilibrium). But external trade plays a negative role: capacity bottlenecks appear in the short and medium runs, then losses in competitiveness, affecting both on imports and exports, reducing the multiplier to 1.2 on average.

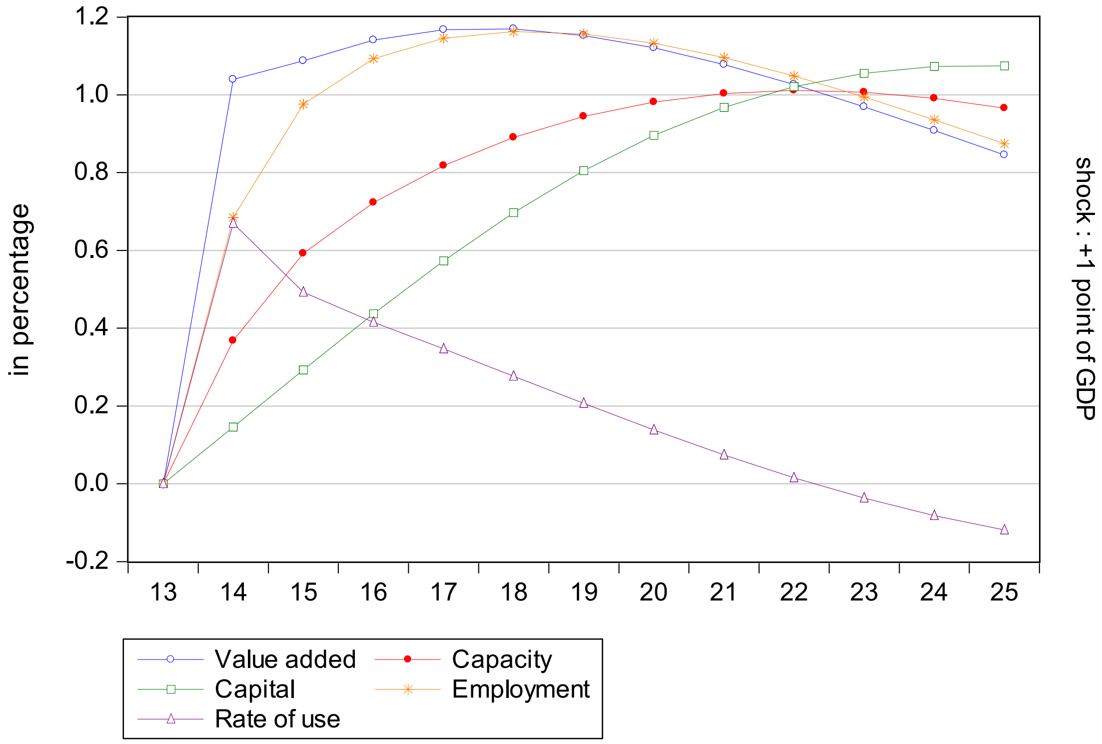
These decreasing and growing effects combine into a rather stable value.

Government demand : Graph no 1 : The supply - demand equilibrium



The second graph shows the adaptation of factors to the new production level.

Government demand : Graph no 2 : The production elements

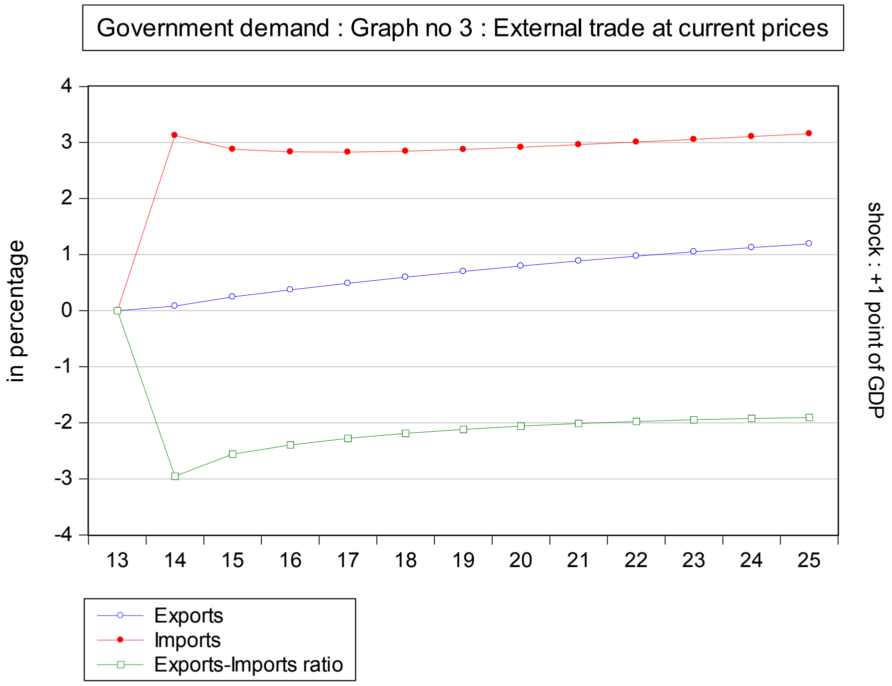


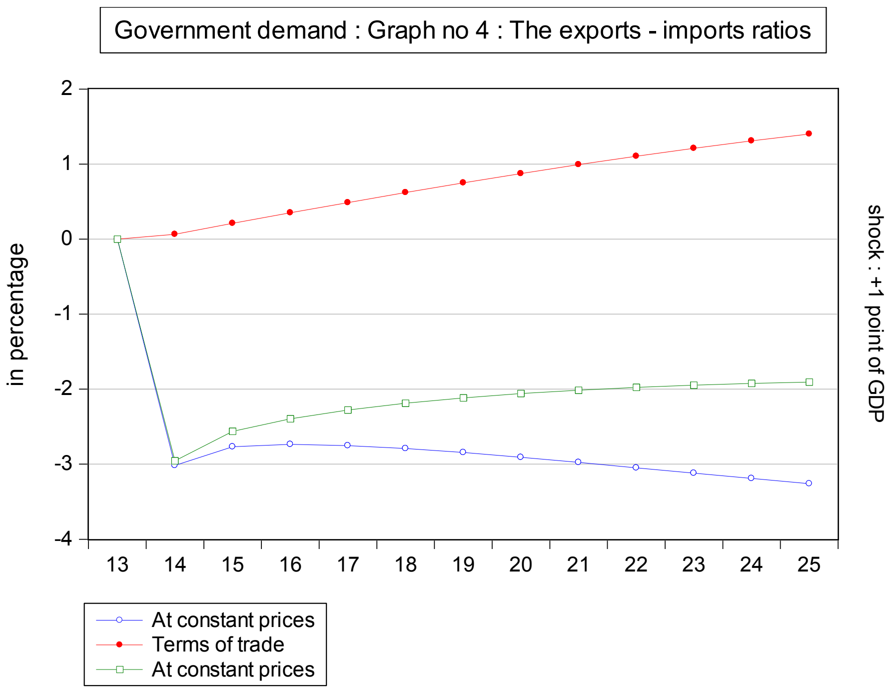
We can see that:

- Employment adapts much faster (following the estimated coefficients).
- In the long run, the rate of use reverts to the baseline value.

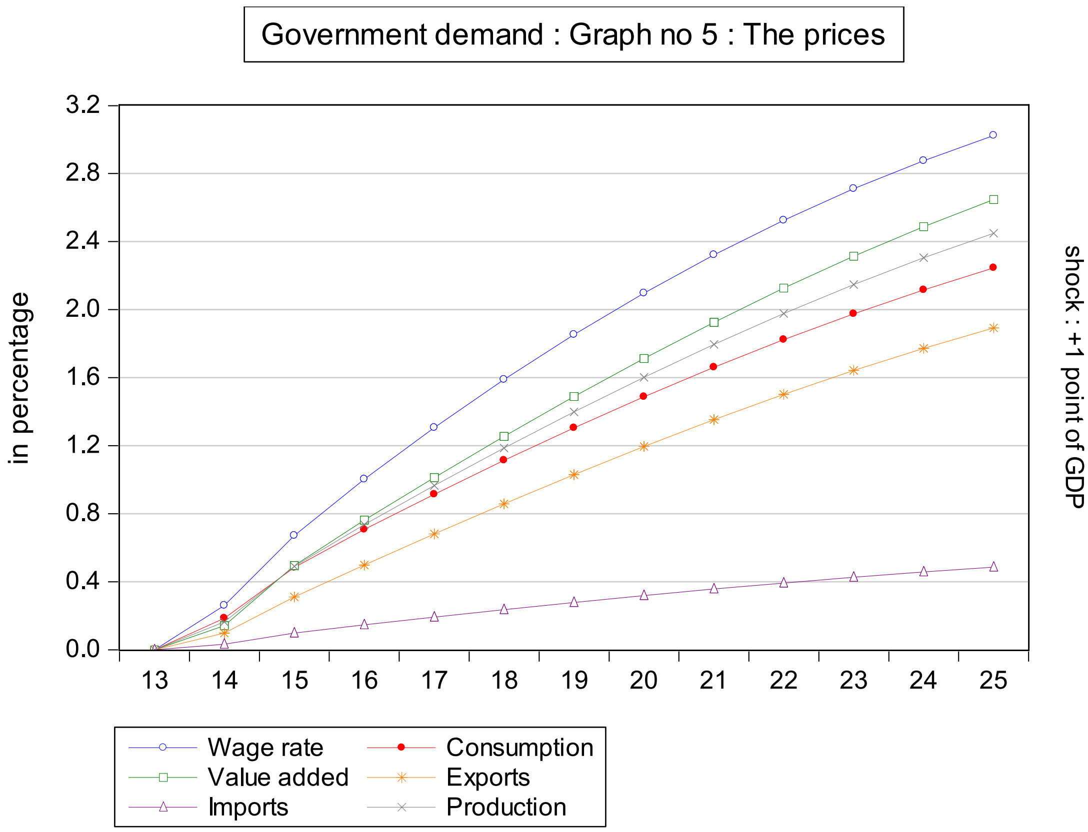
By running the associated programs in the second part, the reader will be able to observe other elements, such as the ex post reduction of the budgetary cost in the short run (due to the various additional tax revenues) followed by an expansion (as the government has to pay the interest on the accumulated debt). But he is also expected to produce his own model, and to observe how his changes (limited or extensive) modify model properties.

Concerning external trade, one can observe that the large increase in the deficit comes almost only from quantities, the terms of trade introducing a small improvement.





Considering the prices, we see that the increase is gradual, and that the ordering is quite logical.



The table on the following page summarizes the evolutions and shows that the government deficit increases as the lower multiplier affects revenue, and the accumulation of debt (the measure is not financed) affects the interest paid.

Table 4: South Africa: An increase in government demand

	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026
Real equlibrium													
GDP	1.06	1.11	1.16	1.19	1.19	1.17	1.14	1.10	1.05	0.99	0.93	0.87	0.80
Final demand	1.89	1.85	1.89	1.92	1.93	1.92	1.91	1.89	1.86	1.82	1.78	1.74	1.70
Productive Investment	1.88	2.03	2.14	2.16	2.13	2.05	1.92	1.78	1.61	1.44	1.26	1.09	0.92
Total Investment	6.04	6.11	6.15	6.15	6.11	6.04	5.95	5.85	5.73	5.61	5.49	5.38	5.26
Household Consumption	0.89	1.08	1.17	1.22	1.26	1.28	1.28	1.28	1.26	1.24	1.21	1.17	1.13
Imports	3.09	2.78	2.68	2.63	2.60	2.59	2.59	2.59	2.61	2.62	2.64	2.66	2.68
Exports	-0.02	-0.07	-0.12	-0.19	-0.26	-0.33	-0.39	-0.46	-0.52	-0.58	-0.64	-0.69	-0.74
Export-import ratio	-3.02	-2.77	-2.74	-2.75	-2.79	-2.84	-2.91	-2.98	-3.05	-3.12	-3.19	-3.26	-3.33
Productive capacity	0.37	0.59	0.72	0.82	0.89	0.94	0.98	1.00	1.01	1.01	0.99	0.97	0.93
Rate of use (*)	0.67	0.49	0.42	0.35	0.28	0.21	0.14	0.07	0.01	-0.04	-0.08	-0.12	-0.15
Deflators													
GDP	0.14	0.49	0.76	1.01	1.25	1.48	1.71	1.92	2.12	2.30	2.48	2.64	2.78
Exports	0.10	0.31	0.50	0.68	0.86	1.03	1.20	1.35	1.50	1.64	1.77	1.89	2.00
Imports	0.03	0.10	0.15	0.19	0.24	0.28	0.32	0.36	0.39	0.43	0.46	0.49	0.51
Consumer price index	0.19	0.48	0.71	0.91	1.11	1.30	1.49	1.66	1.82	1.97	2.12	2.25	2.36
Wage rate	0.26	0.67	1.00	1.31	1.59	1.85	2.10	2.32	2.53	2.71	2.88	3.02	3.15
Export competitiveness	0.10	0.31	0.50	0.68	0.86	1.03	1.20	1.35	1.50	1.64	1.77	1.89	2.00
Import competitiveness	-0.13	-0.39	-0.58	-0.77	-0.94	-1.10	-1.26	-1.41	-1.55	-1.68	-1.81	-1.92	-2.02
Employment													
Firms employment	0.68	0.98	1.09	1.15	1.16	1.16	1.13	1.10	1.05	0.99	0.94	0.87	0.81
Total employment	0.68	0.98	1.09	1.15	1.16	1.16	1.13	1.10	1.05	0.99	0.94	0.87	0.81
Unemployment rate (*)	-0.20	-0.27	-0.29	-0.28	-0.28	-0.27	-0.25	-0.24	-0.22	-0.21	-0.19	-0.18	-0.16
Balances													
Margins rate (*)	0.10	-0.03	-0.08	-0.11	-0.13	-0.15	-0.16	-0.17	-0.17	-0.17	-0.17	-0.16	-0.15
Profits rate (*)	0.52	0.42	0.36	0.31	0.25	0.20	0.15	0.10	0.06	0.02	-0.01	-0.03	-0.04
Government expenditures (*)	1.14	1.37	1.54	1.69	1.81	1.91	2.00	2.07	2.14	2.20	2.25	2.30	2.34
Government revenue (*)	0.41	0.50	0.59	0.66	0.73	0.79	0.85	0.90	0.95	0.99	1.02	1.05	1.08
Government balance (*)	-0.73	-0.87	-0.95	-1.02	-1.08	-1.12	-1.15	-1.17	-1.19	-1.21	-1.23	-1.24	-1.26
(*) In GDP points													

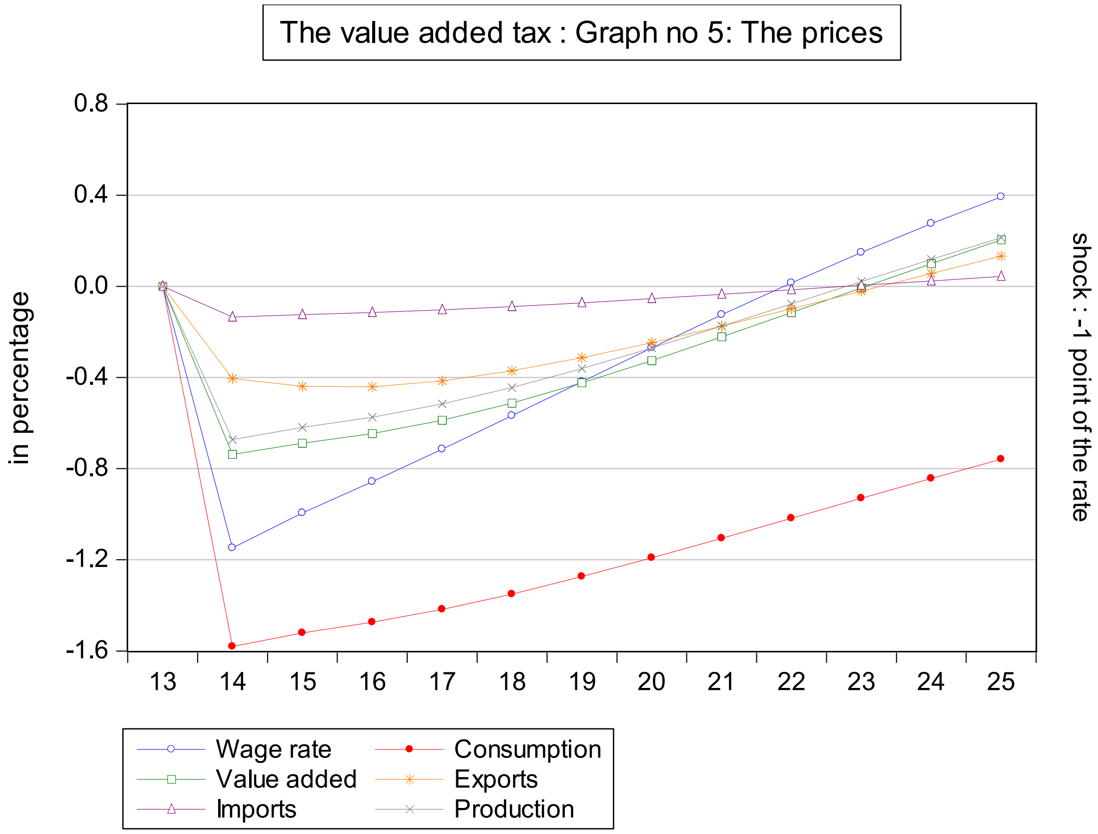
3.5.4.3.2 A decrease in the VAT rate

We move now to a typical supply shock. This shock should not create growth directly.

But both consumption and investment deflators will decrease.

The first will affect wages, partially indexed, thus the wage cost and the value added deflator (excluding taxes). The production price will follow, and the local firms will gain competitiveness in the local and foreign markets, especially the second one as foreign competitors do not benefit from the measure, while imports do.

Also, the lower price of capital will increase profitability.



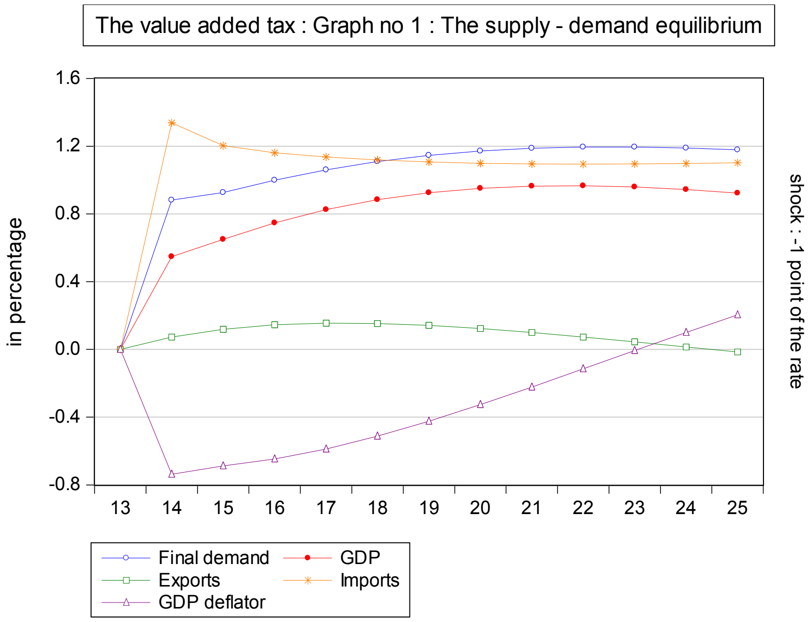
Indeed, the consumption price decreases more than 1%, and the other prices decrease too, including the wage rate in spite of the gains in purchasing power due to the lower unemployment.

The price wage loop extends the disinflation: the growing gains in competitiveness favor exports, and make imports grow less than demand. This adds to the increase in demand: consumption improves, as household revenue profits from the partial indexation of wages on the value added deflator, and the better bargaining power coming from a lower unemployment rate. Investment increases also, but mostly from the adaptation of capacity to a higher production level.

But in the long run, the increase in wages eliminates the deflation of the non-taxed elements.

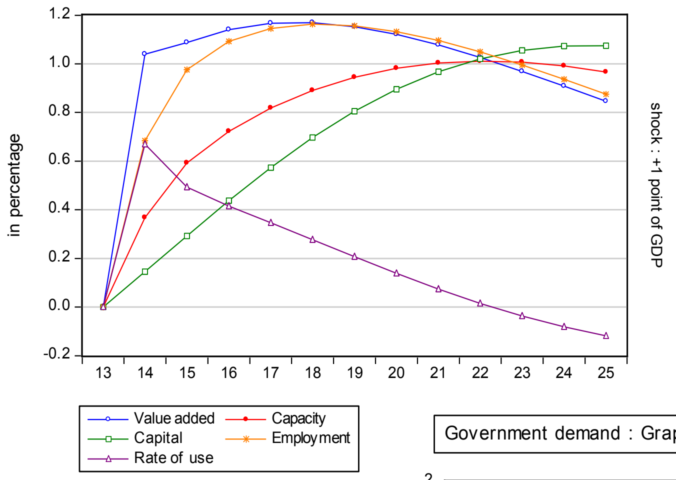
The growing nature of the multiplier (it will stabilize later) delays the adaption of capacities to demand, and the reduction in the increase in the rate of use.

The following graph shows the trade balance at current prices improves over the period, with a growing role of the terms of trade and a decreasing one of the real balance

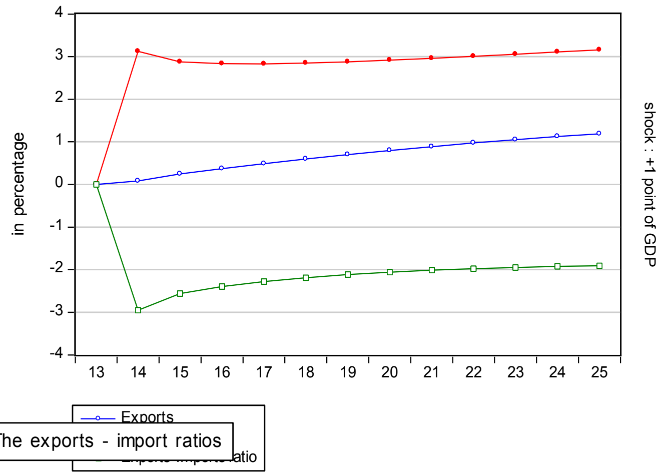


The increase in the government deficit is limited by the lower interest rate (exogenous in real terms). This element can be discussed as the forecast on government interests is not really reliable.

Government demand : Graph no 2 : The production elements



Government demand : Graph no 4 : External trade at current prices



Government demand : Graph no 5 : The exports - import ratios

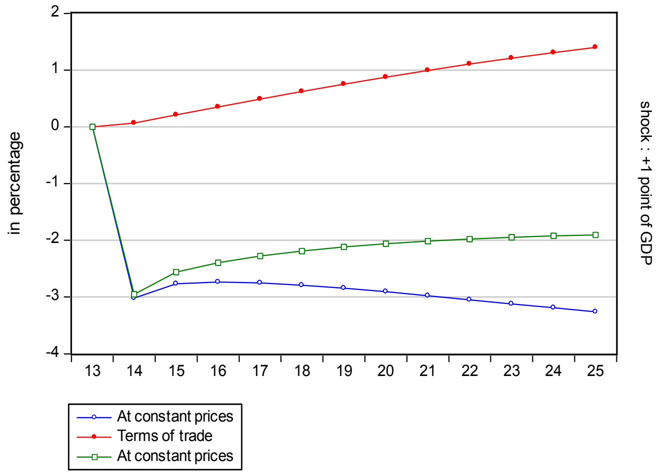


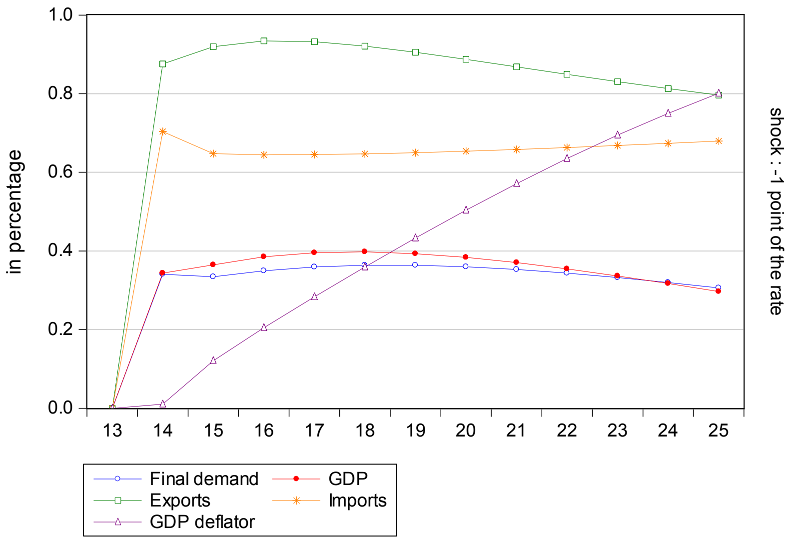
Table 5. South Africa: A decrease in the VAT rate

	2014	2015	2016	2017	2018	2019	2020	2022	2023	2024	2025	2026
Real equlibrium												
GDP	0.56	0.64	0.71	0.77	0.82	0.85	0.86	0.87	0.87	0.86	0.85	0.83
Final demand	0.92	0.94	0.98	1.02	1.06	1.08	1.10	1.12	1.12	1.12	1.12	1.11
Productive Investment	0.92	1.03	1.11	1.16	1.18	1.18	1.17	1.11	1.07	1.03	0.99	0.94
Total Investment	0.57	0.63	0.69	0.72	0.74	0.74	0.73	0.70	0.68	0.65	0.63	0.60
Household Consumption	1.14	1.26	1.35	1.41	1.47	1.51	1.55	1.60	1.61	1.62	1.62	1.63
Imports	1.43	1.26	1.19	1.14	1.12	1.10	1.10	1.10	1.11	1.12	1.13	1.14
Exports	0.05	0.08	0.10	0.10	0.09	0.08	0.06	0.02	-0.00	-0.02	-0.05	-0.07
Export-import ratio	-1.36	-1.16	-1.08	-1.03	-1.01	-1.01	-1.02	-1.06	-1.09	-1.13	-1.17	-1.20
Productive capacity	0.19	0.33	0.42	0.50	0.56	0.61	0.66	0.72	0.74	0.76	0.77	0.77
Rate of use (*)	0.35	0.30	0.28	0.27	0.25	0.22	0.20	0.15	0.12	0.10	0.07	0.05
Deflators												
GDP	-1.56	-1.50	-1.46	-1.41	-1.35	-1.27	-1.19	-1.03	-0.94	-0.86	-0.78	-0.70
Exports	-0.28	-0.29	-0.28	-0.25	-0.21	-0.16	-0.11	0.01	0.07	0.14	0.20	0.26
Imports	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.02	0.01	0.03	0.04	0.06	0.07
Consumer price index	-1.39	-1.33	-1.28	-1.23	-1.17	-1.11	-1.04	-0.90	-0.83	-0.76	-0.70	-0.63
Wage rate	-0.82	-0.69	-0.57	-0.45	-0.33	-0.21	-0.10	0.12	0.22	0.32	0.42	0.50
Export competitiveness	-0.28	-0.29	-0.28	-0.25	-0.21	-0.16	-0.11	0.01	0.07	0.14	0.20	0.26
Import competitiveness	0.38	0.33	0.29	0.25	0.20	0.14	0.08	-0.04	-0.10	-0.17	-0.23	-0.29
Employment												
Firms employment	0.36	0.55	0.66	0.73	0.79	0.82	0.85	0.87	0.87	0.86	0.85	0.83
Total employment	0.36	0.55	0.66	0.73	0.79	0.82	0.85	0.87	0.87	0.86	0.85	0.83
Unemployment rate (*)	-0.11	-0.15	-0.17	-0.18	-0.19	-0.19	-0.19	-0.19	-0.19	-0.18	-0.18	-0.18
Balances												
Margins rate (*)	0.21	0.13	0.09	0.05	0.03	0.00	-0.01	-0.04	-0.05	-0.06	-0.06	-0.06
Profits rate (*)	0.78	0.74	0.71	0.68	0.65	0.63	0.60	0.55	0.52	0.50	0.49	0.47
Government expenditures (*)	-0.90	-0.83	-0.77	-0.72	-0.60	-0.49	-0.40	-0.26	-0.20	-0.14	-0.09	-0.04
Government revenue (*)	-0.98	-0.97	-0.95	-0.92	-0.90	-0.88	-0.85	-0.81	-0.79	-0.77	-0.75	-0.73
Government balance (*)	-0.09	-0.14	-0.17	-0.20	-0.30	-0.38	-0.45	-0.55	-0.59	-0.63	-0.66	-0.68
(*) In GDP points												

3.5.4.3.3 An increase in quotas applied to local exports

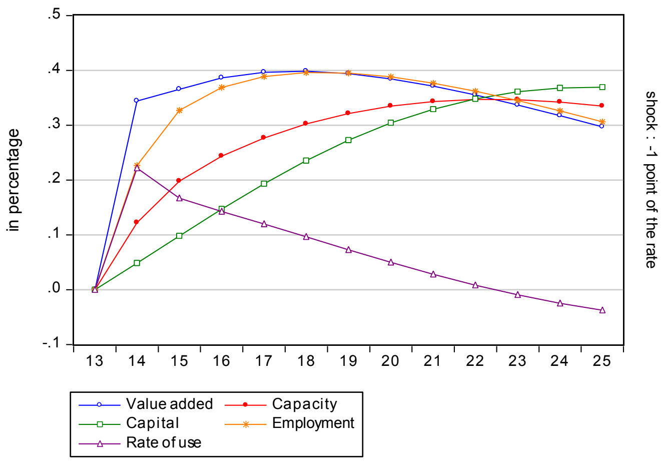
This shock has very similar consequences to the increase of Government demand, except that the initial impulse comes from exports. In the first graph, we see that exports increase by 0.6%, a little less than the ex-ante value (1% multiplied by the 0.6 competitiveness coefficient). Demand contributes also significantly in absolute terms (it is roughly four times higher than exports)

Foreign quotas : Graph no 1 : The supply - demand equilibrium

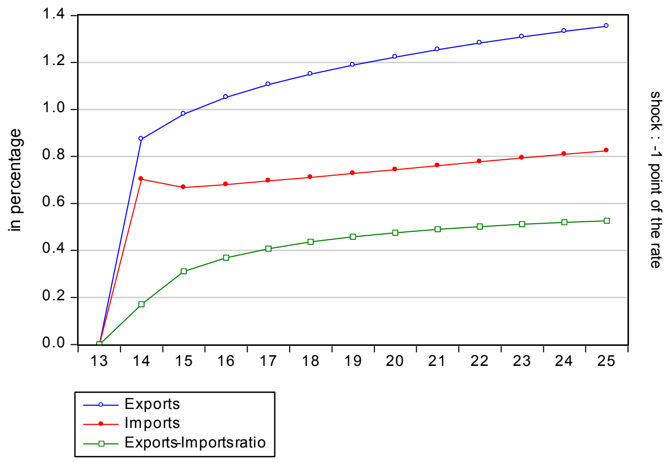


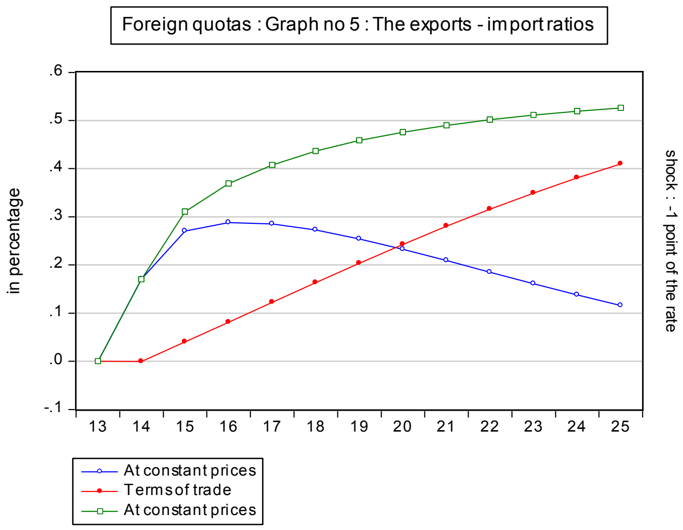
Prices show the usual evolution (the export price is measured excluding prices but to include them one just has to amplify the decrease by 1 point). The trade balance profits both from the real increase and the terms of trade (again, excluding taxes).

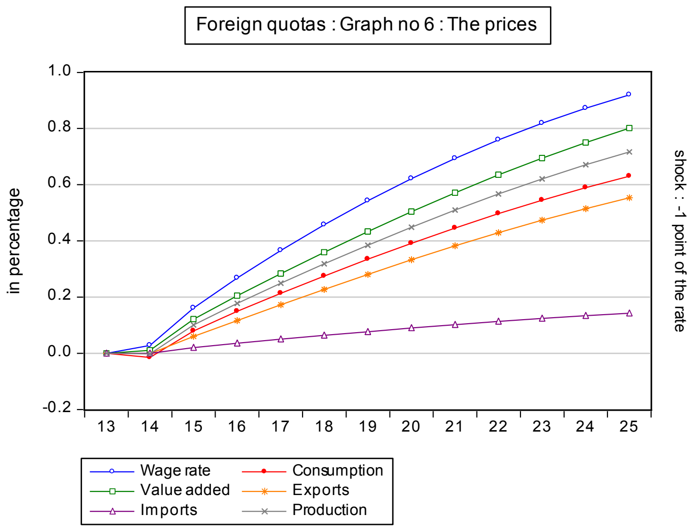
Foreign quotas : Graph no 2 : The production elements



Foreign quotas : Graph no 4 : External trade at current prices







The government no longer has to pay for the shock, so its deficit decreases.

Table 6. South Africa: An increase in foreign quotas

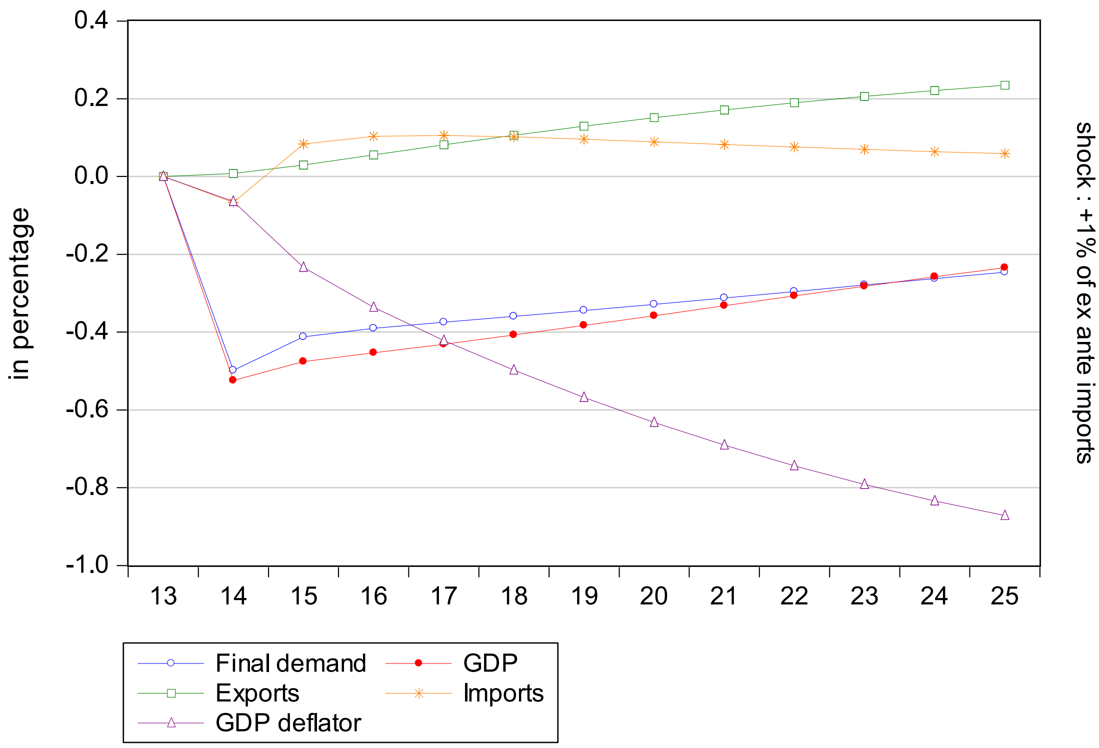
	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026
Real equlibrium													
GDP	0.34	0.36	0.39	0.40	0.40	0.39	0.38	0.37	0.35	0.34	0.32	0.30	0.28
Final demand	0.34	0.33	0.35	0.36	0.36	0.36	0.36	0.35	0.34	0.33	0.32	0.31	0.29
Productive Investment	0.62	0.68	0.72	0.74	0.73	0.70	0.66	0.61	0.56	0.50	0.44	0.39	0.33
Total Investment	0.38	0.42	0.44	0.45	0.45	0.43	0.41	0.38	0.35	0.31	0.28	0.24	0.21
Household Consumption	0.32	0.39	0.42	0.44	0.45	0.46	0.47	0.46	0.46	0.45	0.44	0.43	0.42
Imports	0.70	0.65	0.64	0.65	0.65	0.65	0.65	0.66	0.66	0.67	0.67	0.68	0.69
Exports	0.88	0.92	0.93	0.93	0.92	0.91	0.89	0.87	0.85	0.83	0.81	0.80	0.78
Export-import ratio	0.17	0.27	0.29	0.29	0.27	0.25	0.23	0.21	0.19	0.16	0.14	0.12	0.09
Productive capacity	0.12	0.20	0.24	0.28	0.30	0.32	0.33	0.34	0.35	0.35	0.34	0.33	0.32
Rate of use (*)	0.22	0.17	0.14	0.12	0.10	0.07	0.05	0.03	0.01	-0.01	-0.02	-0.04	-0.05
Deflators													
GDP	0.01	0.12	0.20	0.28	0.36	0.43	0.50	0.57	0.63	0.69	0.75	0.80	0.84
Exports	-0.00	0.06	0.12	0.17	0.23	0.28	0.33	0.38	0.43	0.47	0.51	0.55	0.59
Imports	-0.00	0.02	0.04	0.05	0.06	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15
Consumer price index	-0.01	0.08	0.15	0.21	0.28	0.33	0.39	0.45	0.50	0.54	0.59	0.63	0.67
Wage rate	0.03	0.16	0.27	0.37	0.46	0.54	0.62	0.69	0.76	0.82	0.87	0.92	0.96
Export competitiveness	-0.00	0.06	0.12	0.17	0.23	0.28	0.33	0.38	0.43	0.47	0.52	0.55	0.59
Import competitiveness	0.00	-0.08	-0.14	-0.20	-0.25	-0.31	-0.36	-0.41	-0.45	-0.49	-0.53	-0.57	-0.60
Employment													
Firms employment	0.23	0.33	0.37	0.39	0.40	0.39	0.39	0.38	0.36	0.34	0.33	0.31	0.29
Total employment	0.23	0.33	0.37	0.39	0.40	0.39	0.39	0.38	0.36	0.34	0.33	0.31	0.29
Unemployment rate (*)	-0.07	-0.09	-0.10	-0.10	-0.09	-0.09	-0.09	-0.08	-0.08	-0.07	-0.07	-0.06	-0.06
Balances													
Margins rate (*)	0.04	-0.00	-0.02	-0.03	-0.04	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
Profits rate (*)	0.20	0.16	0.15	0.13	0.11	0.09	0.07	0.06	0.04	0.03	0.02	0.01	0.01
Government expenditures (*)	0.00	0.07	0.11	0.16	0.20	0.23	0.26	0.28	0.30	0.32	0.33	0.34	0.35
Government revenue (*)	0.12	0.15	0.17	0.20	0.22	0.24	0.26	0.27	0.29	0.30	0.31	0.32	0.33
Government balance (*)	0.11	0.08	0.06	0.04	0.02	0.01	0.00	-0.01	-0.01	-0.02	-0.02	-0.02	-0.03
(*) In GDP points													

3.5.4.3.4 An increase in quotas applied to local imports

We are now facing a negative demand shock. Considering the supply – demand equilibrium, the contribution of foreign producers to local demand increases by 1%, so the contribution of local ones decreases by the same absolute amount.

One can be surprised that the ex post increase in imports is so small: this comes from the decrease in local demand and also of the rate of use, which allows limiting losses for the firms who were not able to meet demand (and worked at 100% with a higher potential) in the base simulation.

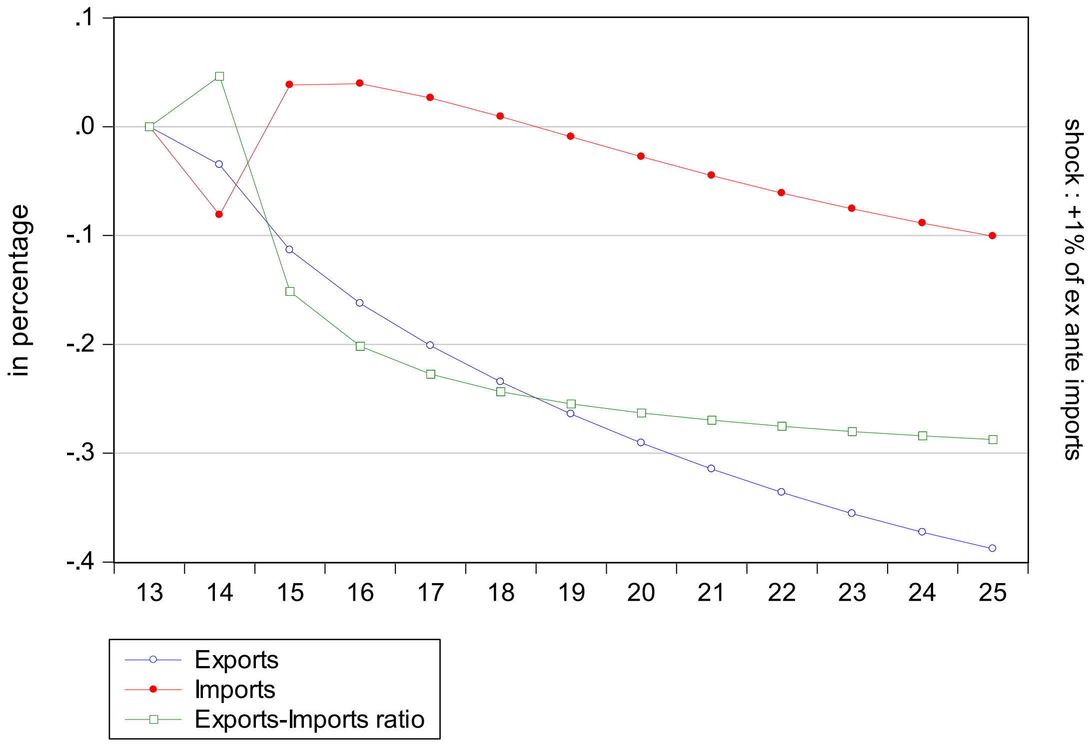
Local quotas : Graph no 1 : The supply - demand equilibrium



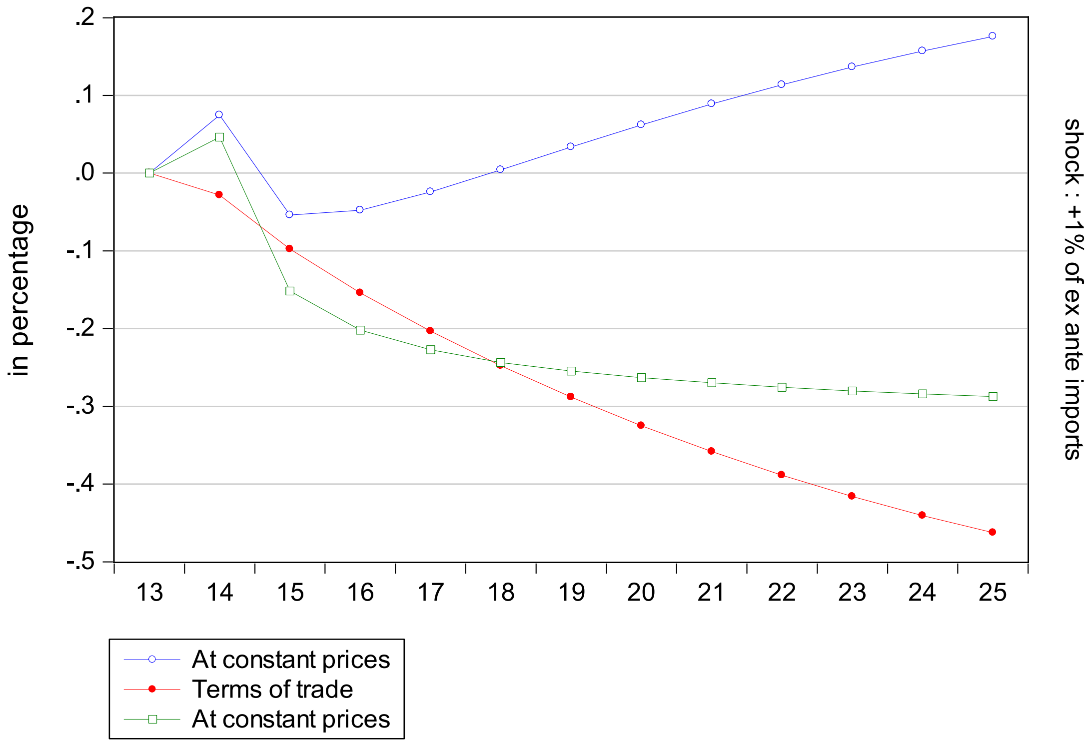
In the long run capacities adapt and only the first effect remains: imports increase.

As to the trade balance, the limitation of the loss in real terms is paid by a further loss on the terms of trade: in the long run, both effects present roughly the same size.

Local quotas : Graph no 4 : External trade at current prices



Local quotas : Graph no 5 : The exports - import ratios



The table on the next page shows that if the government suffers no loss ex-ante, the decrease in activity and revenue deteriorates the balance.

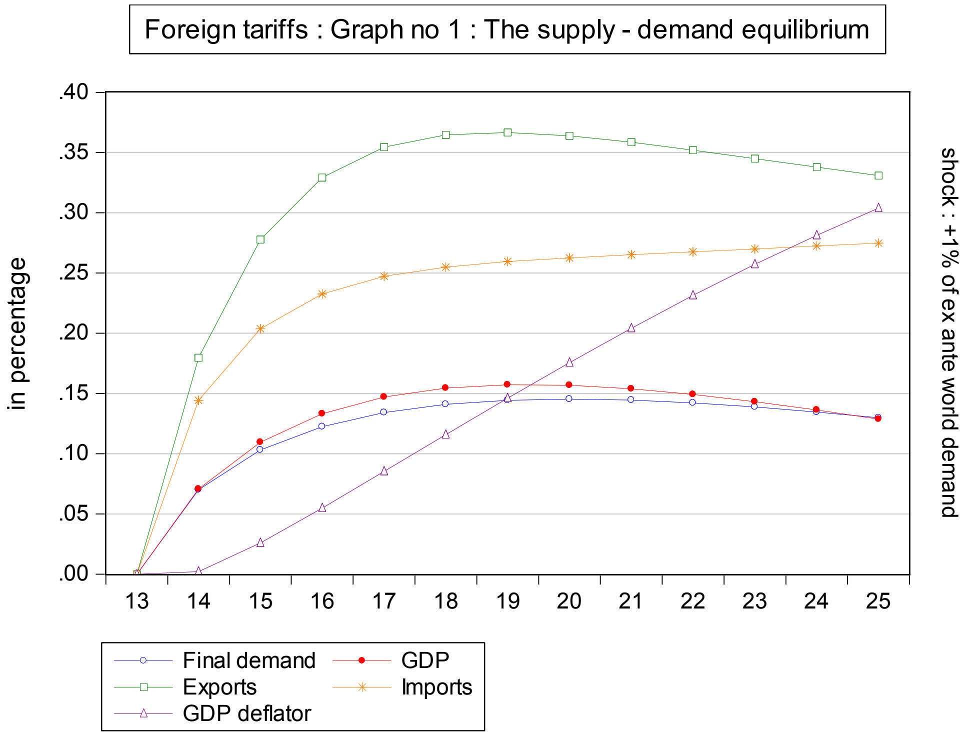
Table 7. South Africa: An increase in local quotas

	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2026
Real equlibrium												
GDP	-0.51	-0.50	-0.50	-0.49	-0.48	-0.47	-0.46	-0.44	-0.42	-0.40	-0.38	-0.34
Final demand	-0.48	-0.42	-0.41	-0.41	-0.41	-0.40	-0.39	-0.39	-0.38	-0.37	-0.36	-0.34
Productive Investment	-0.87	-0.81	-0.78	-0.74	-0.69	-0.64	-0.59	-0.54	-0.50	-0.45	-0.41	-0.34
Total Investment	-0.53	-0.50	-0.48	-0.46	-0.43	-0.40	-0.37	-0.34	-0.31	-0.29	-0.26	-0.21
Household Consumption	-0.45	-0.50	-0.52	-0.52	-0.53	-0.53	-0.53	-0.53	-0.52	-0.51	-0.50	-0.48
Imports	-0.04	0.12	0.16	0.17	0.18	0.18	0.17	0.16	0.15	0.14	0.13	0.10
Exports	0.01	0.03	0.05	0.07	0.10	0.12	0.15	0.17	0.19	0.21	0.23	0.27
Export-import ratio	0.05	-0.09	-0.11	-0.10	-0.08	-0.05	-0.02	0.01	0.04	0.07	0.11	0.17
Productive capacity	-0.18	-0.28	-0.32	-0.35	-0.37	-0.38	-0.39	-0.39	-0.39	-0.38	-0.37	-0.35
Rate of use (*)	-0.33	-0.22	-0.18	-0.15	-0.12	-0.09	-0.07	-0.05	-0.03	-0.02	-0.00	0.02
Deflators												
GDP	-0.05	-0.20	-0.30	-0.39	-0.47	-0.55	-0.63	-0.70	-0.77	-0.83	-0.89	-1.00
Exports	-0.04	-0.12	-0.19	-0.26	-0.32	-0.38	-0.44	-0.49	-0.54	-0.59	-0.63	-0.71
Imports	-0.01	-0.04	-0.06	-0.07	-0.09	-0.10	-0.12	-0.13	-0.14	-0.15	-0.16	-0.18
Consumer price index	-0.07	-0.19	-0.27	-0.34	-0.40	-0.47	-0.53	-0.58	-0.64	-0.69	-0.74	-0.82
Wage rate	-0.10	-0.27	-0.39	-0.49	-0.59	-0.68	-0.76	-0.83	-0.90	-0.97	-1.02	-1.12
Export competitiveness	-0.04	-0.12	-0.19	-0.26	-0.32	-0.38	-0.44	-0.49	-0.54	-0.59	-0.63	-0.71
Import competitiveness	0.05	0.16	0.23	0.29	0.35	0.41	0.47	0.52	0.57	0.62	0.66	0.74
Employment												
Firms employment	-0.34	-0.46	-0.49	-0.49	-0.49	-0.48	-0.46	-0.45	-0.43	-0.41	-0.39	-0.35
Total employment	-0.34	-0.46	-0.49	-0.49	-0.49	-0.48	-0.46	-0.45	-0.43	-0.41	-0.39	-0.35
Unemployment rate (*)	0.10	0.13	0.13	0.12	0.11	0.11	0.10	0.10	0.09	0.08	0.08	0.07
Balances												
Margins rate (*)	-0.06	0.01	0.03	0.04	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.05
Profits rate (*)	-0.26	-0.20	-0.16	-0.14	-0.12	-0.10	-0.08	-0.07	-0.06	-0.05	-0.04	-0.04
Government expenditures (*)	-0.06	-0.14	-0.19	-0.24	-0.27	-0.30	-0.32	-0.34	-0.36	-0.38	-0.39	-0.42
Government revenue (*)	-0.17	-0.19	-0.22	-0.24	-0.26	-0.28	-0.30	-0.31	-0.33	-0.34	-0.35	-0.37
Government balance (*)	-0.11	-0.05	-0.02	-0.00	0.01	0.02	0.03	0.03	0.04	0.04	0.04	0.05
(*) In GDP points												

3.5.4.3.5 A decrease in the foreign tariffs rate

As we do not consider the decrease in world inflation coming from lower tariffs on South African products, the results are very similar to the shock on quotas (a typical demand shock).

The improved competitiveness will increase the demand addressed to South Africa, with basically the same effects on GDP as Government demand, from the supply – demand equilibrium.

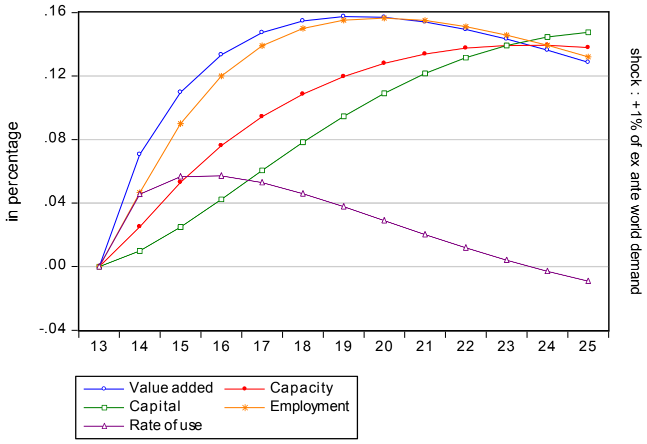


Of course lower import prices from South Africa will reduce inflation in the rest of the world, which should affect local import prices. World GDP should also change (in a dubious way, positive through disinflation, negative through the higher share of France in world trade). But both these effects can be considered negligible, especially if we consider the cost of the alternative: building a reliable world model.

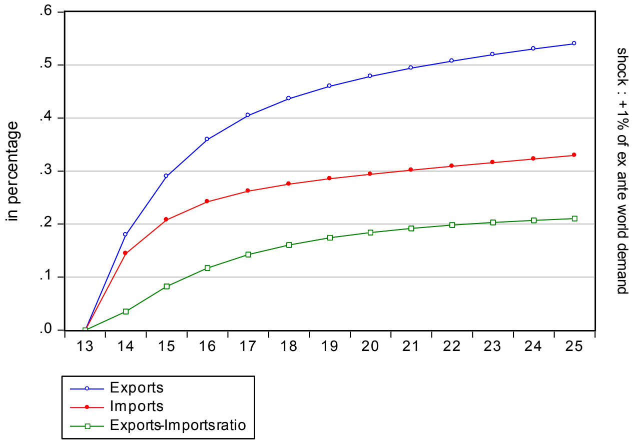
It is interesting to observe that the increase in local activity inverts the gain in trade at constant prices in the long run, but not as current ones as this loss is due to local inflation, which "improves" also the terms of trade.

On the whole, the current gain stabilizes after a while, when capacities have adapted to the new demand level.

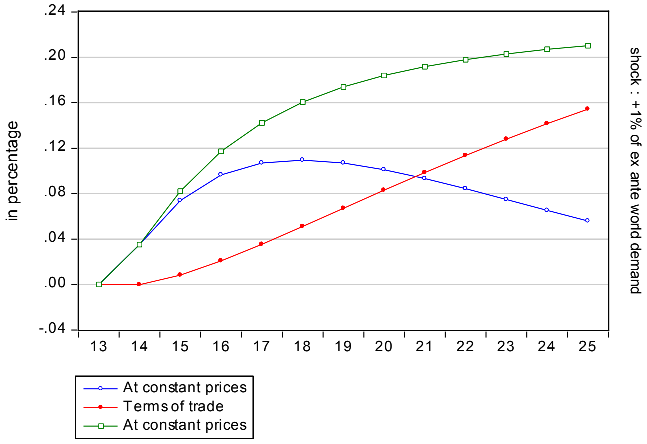
Foreign tariffs : Graph no 2 : The production elements



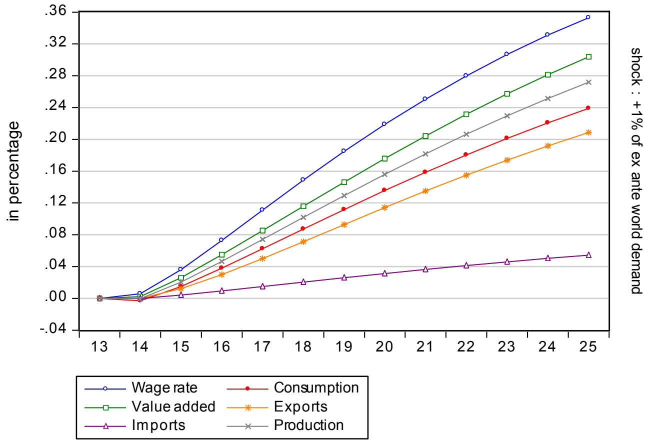
Foreign tariffs : Graph no 4 : External trade at current prices



Foreign tariffs : Graph no 5 : The exports - import ratios



Foreign tariffs : Graph no 6 : The prices



This shock traditionally brings the most complex mechanisms, and its consequences are quite volatile from one model to the other. Two main channels have to be considered:

- The improved imports competitiveness increases their share in local demand, reducing local output, with the traditional consequences of a demand shock, only negative.
- The lower import prices bring global disinflation, especially for demand but also for value added through lower wages (partially indexed on the consumer price). This disinflation helps local firms to compete with foreign producers on the local and foreign markets, with a reduction in the initial gap on the first and a gain on the second.

Local tariffs : Graph no 1 : The supply - demand equilibrium

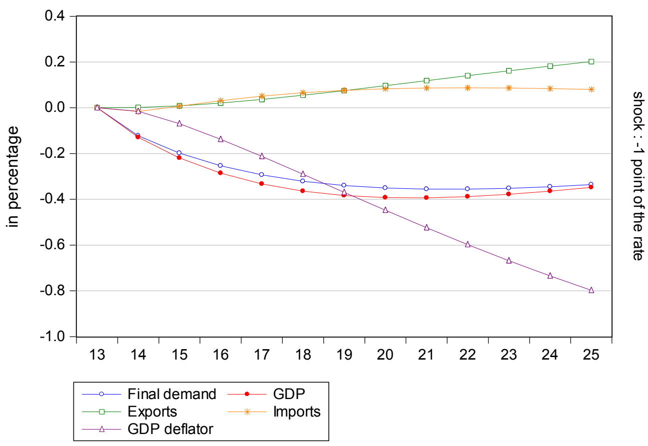


Table 8. South Africa: A decrease in local tariffs

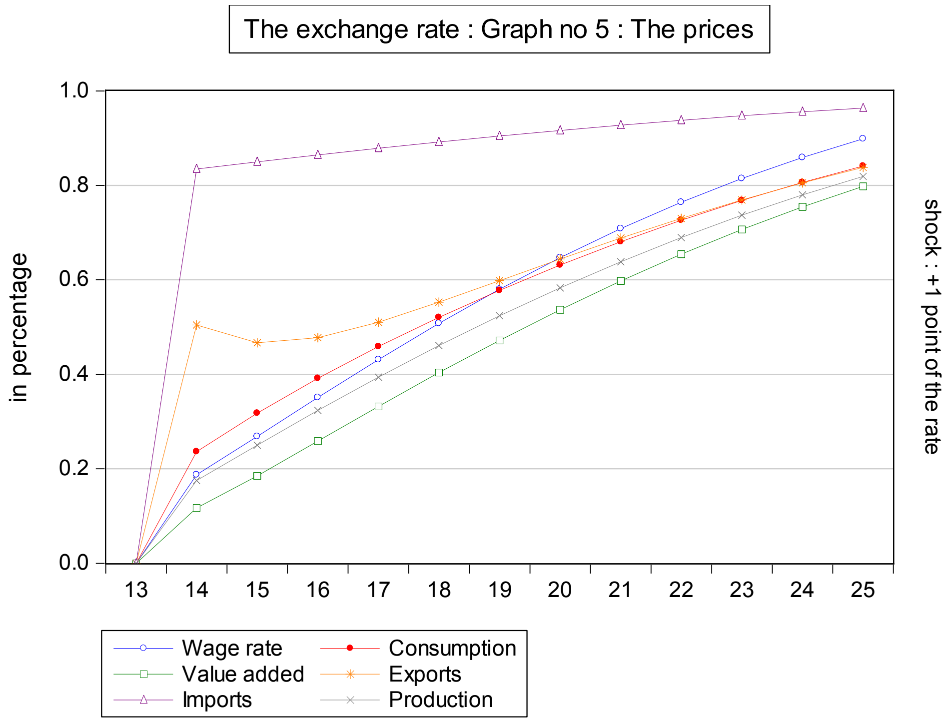
	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
Real equilibrium												
GDP	-0.13	-0.22	-0.28	-0.32	-0.35	-0.36	-0.37	-0.37	-0.37	-0.36	-0.35	-0.34
Final demand	-0.12	-0.19	-0.24	-0.27	-0.29	-0.31	-0.32	-0.32	-0.32	-0.32	-0.32	-0.31
Productive Investment	-0.22	-0.35	-0.44	-0.50	-0.52	-0.53	-0.52	-0.50	-0.48	-0.45	-0.42	-0.39
Total Investment	-0.13	-0.22	-0.27	-0.31	-0.32	-0.33	-0.33	-0.32	-0.30	-0.29	-0.27	-0.25
Household Consumption	-0.11	-0.20	-0.27	-0.33	-0.36	-0.39	-0.41	-0.42	-0.43	-0.43	-0.43	-0.43
Imports	-0.01	0.02	0.05	0.08	0.10	0.12	0.12	0.13	0.13	0.12	0.12	0.11
Exports	0.00	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.12	0.14	0.15	0.17
Export-import ratio	0.01	-0.01	-0.04	-0.05	-0.06	-0.05	-0.04	-0.03	-0.01	0.01	0.03	0.06
Productive capacity	-0.05	-0.10	-0.15	-0.20	-0.23	-0.26	-0.28	-0.29	-0.31	-0.31	-0.31	-0.31
Rate of use (*)	-0.08	-0.11	-0.13	-0.13	-0.12	-0.11	-0.09	-0.08	-0.07	-0.05	-0.04	-0.02
Deflators												
GDP	-0.01	-0.06	-0.12	-0.18	-0.25	-0.31	-0.38	-0.44	-0.51	-0.57	-0.63	-0.68
Exports	-0.01	-0.04	-0.07	-0.12	-0.16	-0.21	-0.26	-0.30	-0.35	-0.39	-0.44	-0.48
Imports	-0.00	-0.01	-0.02	-0.03	-0.05	-0.06	-0.07	-0.08	-0.09	-0.10	-0.11	-0.13
Consumer price index	-0.02	-0.06	-0.11	-0.16	-0.21	-0.27	-0.32	-0.37	-0.42	-0.47	-0.52	-0.56
Wage rate	-0.02	-0.08	-0.16	-0.23	-0.31	-0.39	-0.47	-0.54	-0.61	-0.67	-0.73	-0.79
Export competitiveness	-0.01	-0.04	-0.07	-0.12	-0.16	-0.21	-0.26	-0.30	-0.35	-0.39	-0.44	-0.48
Import competitiveness	-0.95	-0.92	-0.87	-0.83	-0.78	-0.73	-0.68	-0.64	-0.59	-0.55	-0.51	-0.47
Employment												
Firms employment	-0.08	-0.17	-0.25	-0.30	-0.33	-0.36	-0.37	-0.37	-0.37	-0.37	-0.36	-0.34
Total employment	-0.08	-0.17	-0.25	-0.30	-0.33	-0.36	-0.37	-0.37	-0.37	-0.37	-0.36	-0.34
Unemployment rate (*)	0.03	0.05	0.07	0.08	0.08	0.09	0.09	0.09	0.08	0.08	0.08	0.07
Balances												
Margins rate (*)	-0.01	-0.01	0.00	0.01	0.02	0.03	0.04	0.04	0.04	0.05	0.05	0.05
Profits rate (*)	-0.07	-0.10	-0.11	-0.11	-0.11	-0.10	-0.10	-0.09	-0.08	-0.07	-0.06	-0.05
Government expenditures (*)	-0.01	-0.04	-0.07	-0.10	-0.13	-0.16	-0.18	-0.21	-0.23	-0.24	-0.26	-0.27
Government revenue (*)	-0.37	-0.41	-0.45	-0.48	-0.51	-0.53	-0.56	-0.58	-0.59	-0.61	-0.62	-0.63
Government balance (*)	-0.36	-0.38	-0.38	-0.38	-0.38	-0.37	-0.37	-0.37	-0.37	-0.37	-0.36	-0.36

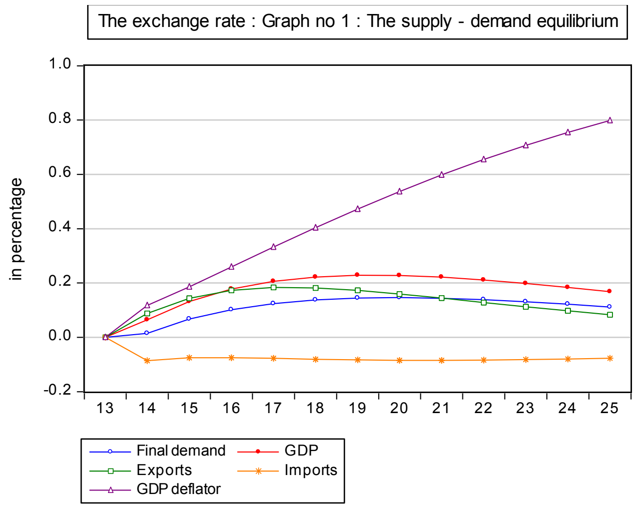
(*) In GDP points

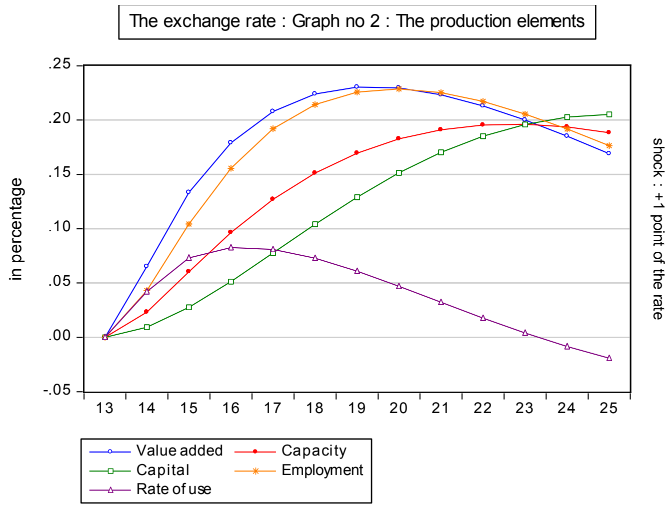
3.5.4.4 An Evaluation

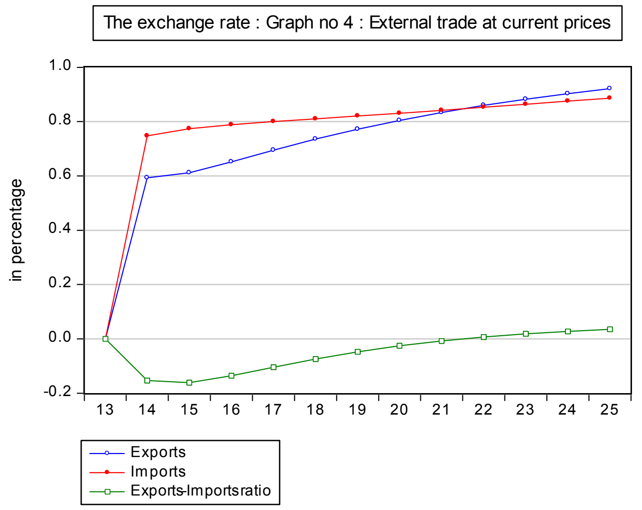
We will depreciate the Rand by 1%.

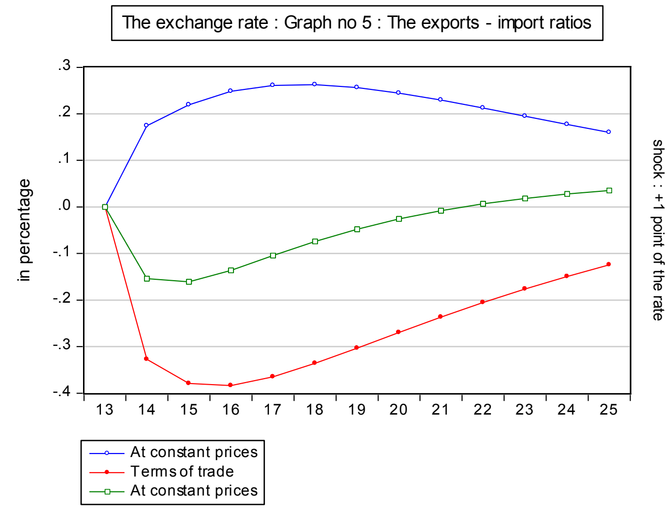
Our first graph presents prices, the second quantities. Concerning the first, we guess that in the long run they all increase by 1%. But the increase is slow, and presents some overshooting in the medium run (due to the inertia coming from low error correcting coefficients). Logically, the imports (essentially) and exports deflators are the fastest to take the decision into account.











Concerning quantities, exports profit the most from the shock, improving GDP. But imports actually increase a little, the gains in competitiveness being more than balanced by the additional demand (not only from local investment and consumption, but also from exports which call for importing intermediary goods).

In the long run, the full adaptation of inflation will make all effects disappear, after a temporary reversal due to the overshooting of local deflators. But this is not yet the case on our 12 year period (one can easily control that it would be if the correcting coefficients were higher).

The last graph presents the evolution of trade. We see that the gains at current prices are very small, as the improvement at constant prices is roughly compensated by a symmetric loss on the terms of trade. Of course, in the long term, all the values will converge to zero.

Table 9. South Africa: A devaluation of the Rand

	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
Real equilibrium												
GDP	0.06	0.12	0.17	0.20	0.21	0.22	0.22	0.22	0.21	0.20	0.19	0.18
Final demand	0.00	0.05	0.08	0.10	0.11	0.12	0.12	0.12	0.12	0.11	0.11	0.10
Productive Investment	0.10	0.21	0.27	0.31	0.32	0.32	0.31	0.29	0.27	0.25	0.22	0.20
Total Investment	0.06	0.13	0.17	0.19	0.20	0.20	0.19	0.18	0.17	0.16	0.14	0.13
Household Consumption	-0.04	0.01	0.06	0.09	0.11	0.13	0.13	0.14	0.14	0.14	0.13	0.13
Imports	-0.11	-0.11	-0.11	-0.12	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.12	-0.12
Exports	0.09	0.15	0.18	0.20	0.20	0.19	0.18	0.17	0.16	0.15	0.13	0.12
Export-import ratio	0.20	0.26	0.30	0.32	0.33	0.33	0.32	0.31	0.29	0.27	0.26	0.24
Productive capacity	0.02	0.06	0.09	0.12	0.14	0.16	0.17	0.17	0.18	0.18	0.18	0.17
Rate of use (*)	0.04	0.07	0.08	0.08	0.07	0.07	0.05	0.04	0.03	0.02	0.01	0.00
Deflators												
GDP	0.09	0.14	0.20	0.26	0.32	0.38	0.43	0.49	0.54	0.58	0.63	0.67
Exports	0.49	0.44	0.44	0.46	0.50	0.53	0.57	0.61	0.64	0.68	0.71	0.74
Imports	0.83	0.84	0.85	0.87	0.88	0.89	0.90	0.91	0.92	0.92	0.93	0.94
Consumer price index	0.21	0.28	0.34	0.40	0.45	0.50	0.54	0.58	0.62	0.66	0.70	0.73
Wage rate	0.14	0.21	0.27	0.34	0.40	0.46	0.52	0.57	0.62	0.67	0.71	0.75
Export competitiveness	-0.51	-0.55	-0.55	-0.53	-0.50	-0.46	-0.43	-0.39	-0.35	-0.32	-0.29	-0.26
Import competitiveness	0.68	0.63	0.58	0.53	0.49	0.45	0.41	0.37	0.34	0.30	0.27	0.24
Employment												
Firms employment	0.04	0.10	0.15	0.18	0.20	0.22	0.22	0.22	0.21	0.21	0.20	0.18
Total employment	0.04	0.10	0.15	0.18	0.20	0.22	0.22	0.22	0.21	0.21	0.20	0.18
Unemployment rate (*)	-0.01	-0.03	-0.04	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.04	-0.04	-0.04
Balances												
Margins rate (*)	-0.01	-0.02	-0.02	-0.03	-0.03	-0.03	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04
Profits rate (*)	-0.04	-0.02	-0.01	-0.01	-0.01	-0.02	-0.02	-0.03	-0.03	-0.04	-0.04	-0.05
Government expenditures (*)	0.15	0.19	0.23	0.26	0.28	0.30	0.31	0.32	0.33	0.34	0.35	0.36
Government revenue (*)	0.05	0.09	0.12	0.14	0.16	0.18	0.19	0.21	0.22	0.23	0.24	0.25
Government balance (*)	-0.09	-0.11	-0.12	-0.13	-0.12	-0.12	-0.12	-0.12	-0.11	-0.11	-0.11	-0.11

3.5.5 Stochastic Simulations

Until now, we have only considered simulations in which the residual was supposedly known, either with a zero value (the most probable) or with a value decided by the user. As estimation statistics give use an estimate of residual distribution, we can also perform stochastic simulations, in which the residual is drawn randomly. This also applies to coefficients, which can be drawn around their estimated value.

This will allow us to define the precision of the model, and in particular a confidence interval.

Of course, the model is also subject to two other type of errors:

- The fact that the estimated formula itself could be wrong (for a given behavior, many formulas will pass all tests), or that between the sample period and the future, the agents have modified their behavior.
- The error on the assumptions used for the forecast, for which the model should not be accounted responsible.

We will not consider these last types.

- The error on residuals can be measured by drawing at random a sample of residuals, undertaking the associated projection, and observing the statistical characteristics of results (a « Monte Carlo » technique). The drawing can either use a normal law with the estimated characteristics, or by drawing randomly a sample from the sequence of observed residuals (with or without putting back the elements selected into the pool). This technique is "bootstrapping".
- The same type of method applies to the uncertainty due to coefficients:

Three main error types should be considered:

- The bias: in the case of a model with non - linear properties, the mean of the solution will not be identical to a zero residual.
- The difference comes of course from the non-linearities of the equations, and any economic model presents non-linearities. Obvious cases are the presence of variables at current prices, product of a variable at constant prices by a deflator, or variables computed by applying a growth rate to their past value.
- The standard error: this criterion will assess the reliability of results (evaluating a confidence interval, or range of possible values.
- The distribution: what we want is a graph of the probability distribution of the random solutions.

If we consider the error on coefficients, the process is a little more complex. It might be necessary to take into account the correlation between coefficients, and the drawing will call for a multivariate normal law, with a non-diagonal covariance matrix.

3.5.5.1 Application to our model

We shall produce two stochastic simulations of our model, including or not coefficient uncertainty. The period used will be 2013-2025, and we shall produce 10 000 replications, a figure quite manageable for such a small model (it actually took 10 seconds on a rather powerful computer). The software failed to reach the solution 4 times[31].

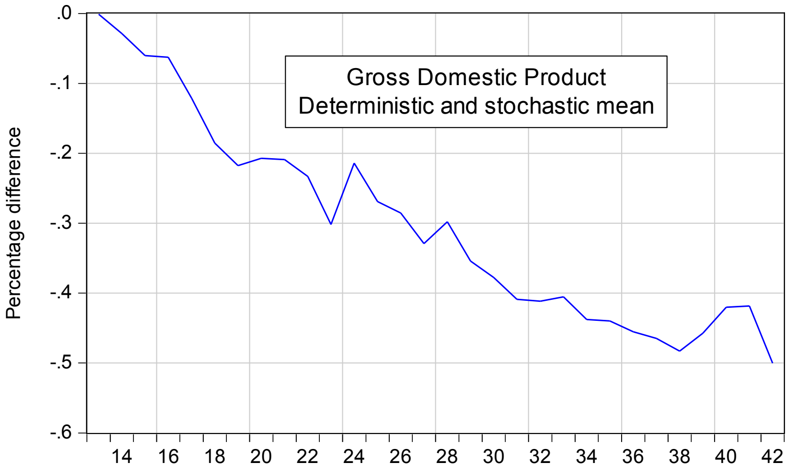
We shall consider:

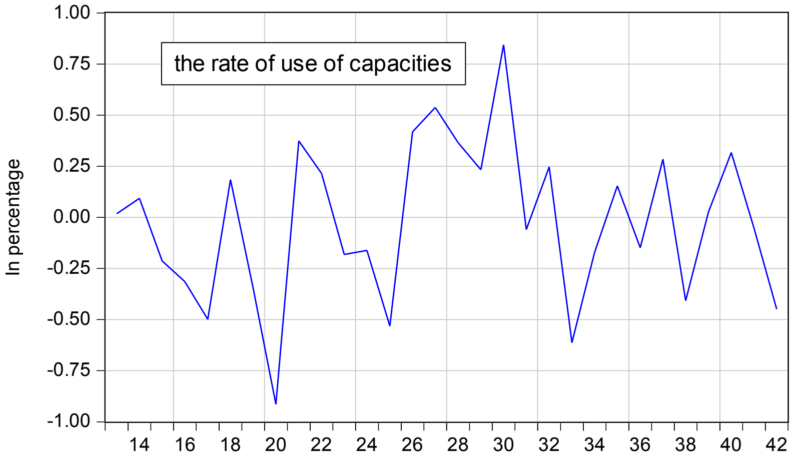
- The bias.
- The standard deviation.
- Confidence intervals.
- The distribution of the results.

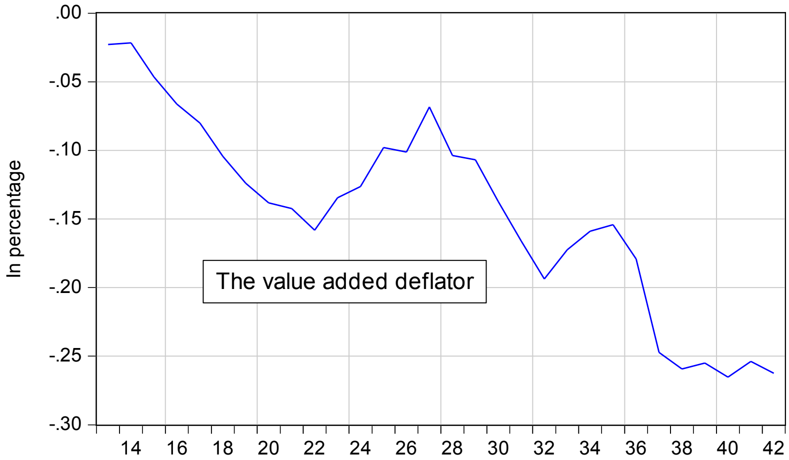
3.5.5.2 The Bias

The following graphs presents the ratio between the deterministic solution and the mean of the replications.

[31] This does not mean there was no solution.







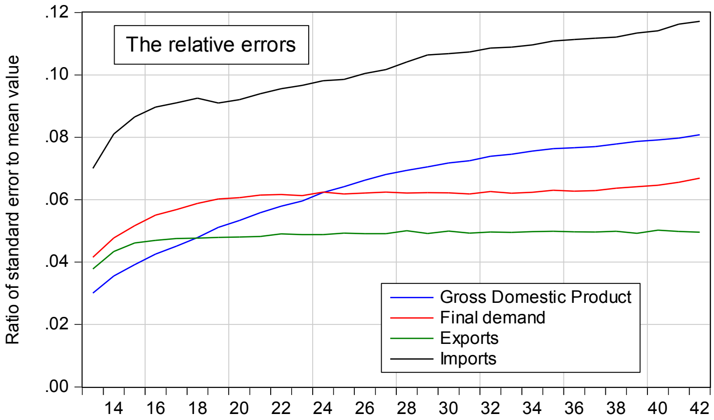
For variables with dimension, we can observe a regularly growing bias, not very large but significant (around 0.01% per year for prices, 0.02% per year for GDP.

This bias might be due to the low value of the correcting term in the price and wage equations. We have seen that for our forecasts we have calibrated these values.

3.5.5.3 The Standard Deviations

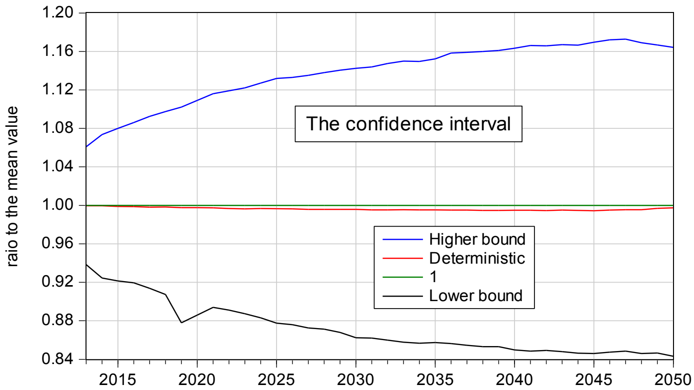
We will now present the standard deviations, as a ratio to the mean value of the variable.

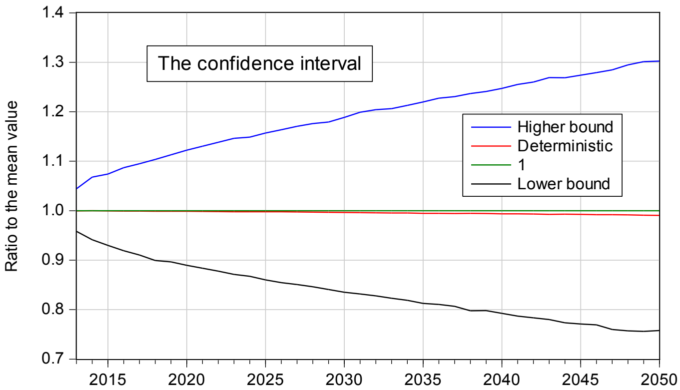
We can see that the relative error stabilizes for quantities (slower for GDP) but that for prices it will take much more time.



3.5.5.4 The Confidence Intervals

We observe that the confidence interval for GDP stabilizes at a level about three times higher than the initial value, and for the deflator it keeps widening (it will stabilize much later).

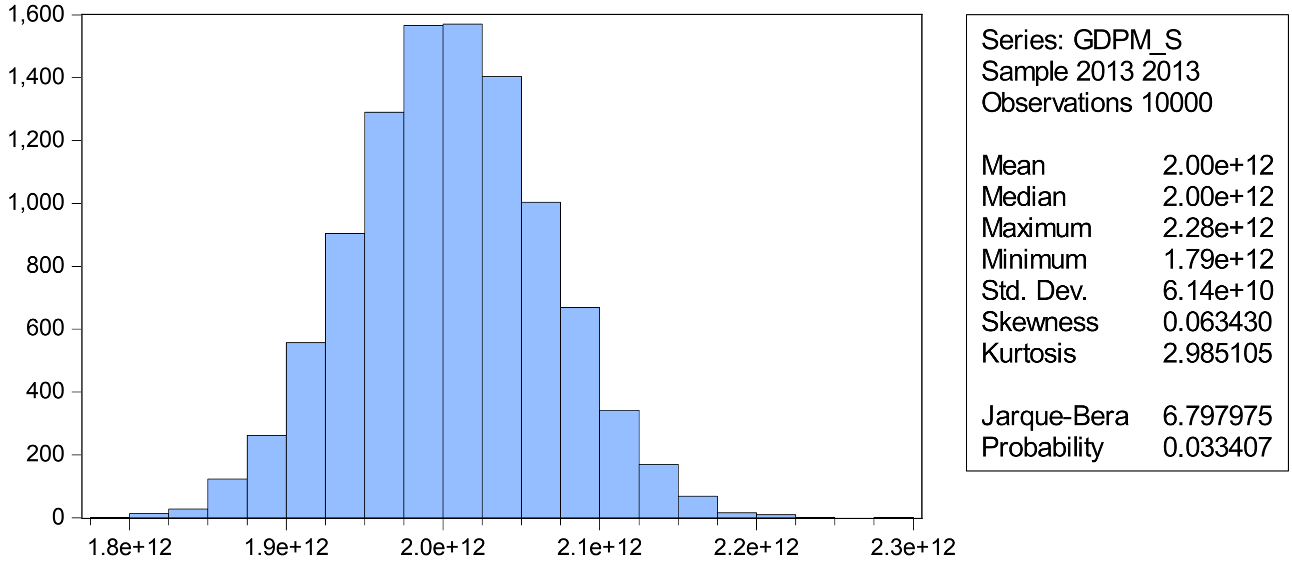




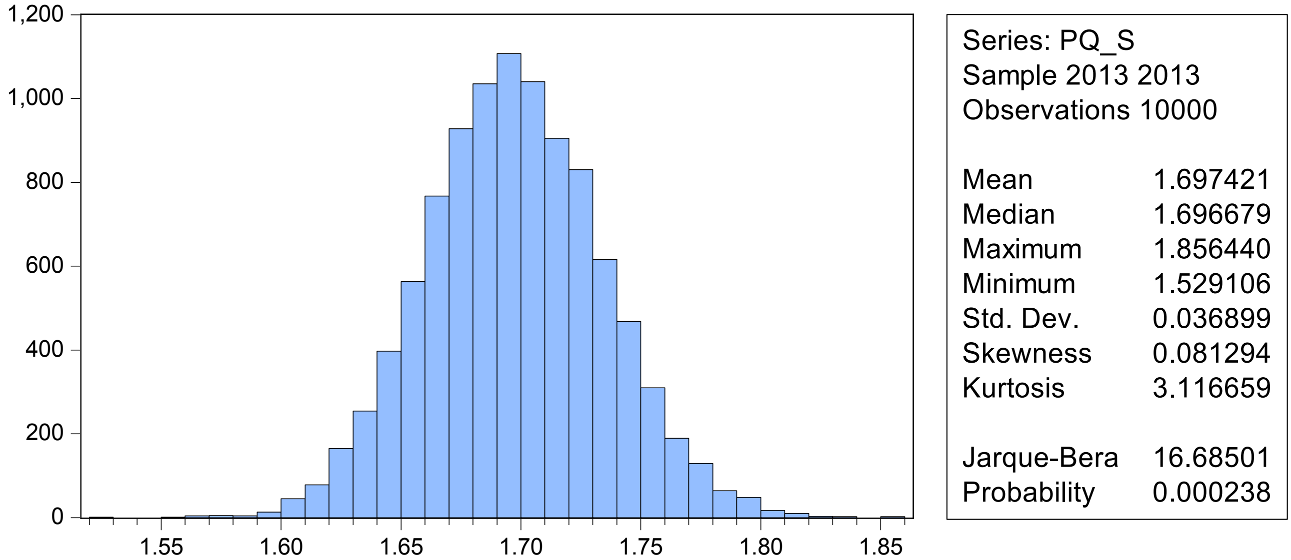
3.5.5.5 The Distribution

For the same variables, we see that the distribution is a little skewed (the criterion is positive but low) and follows rather well a normal distribution (the kurtosis is very close to 3).

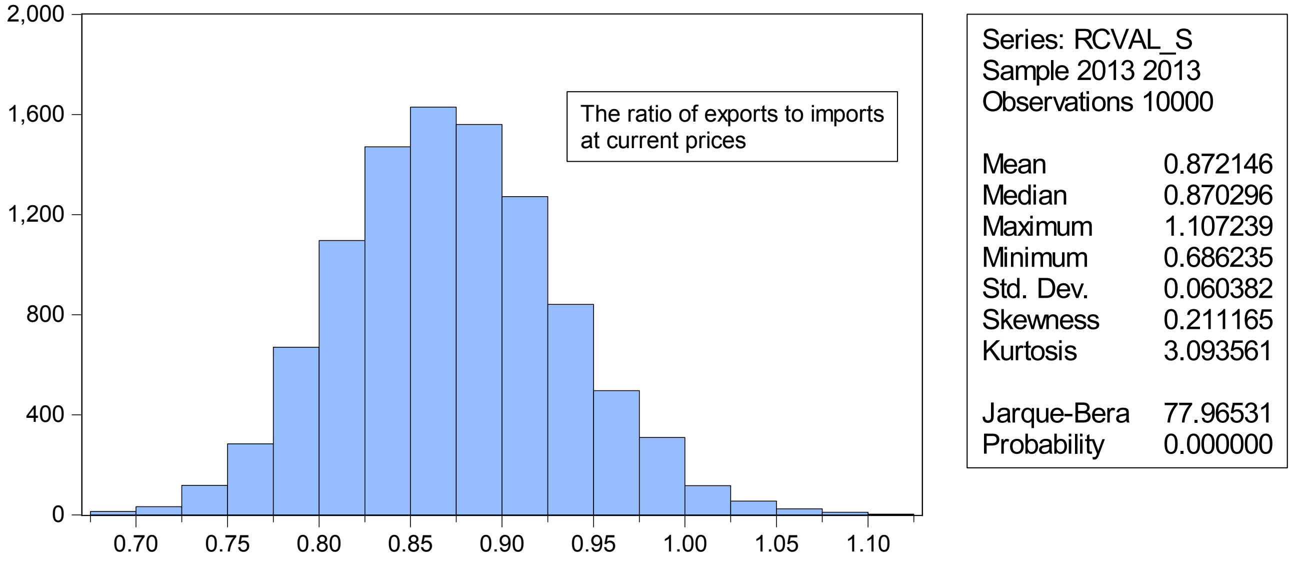
The results for GDP:



For the value-added deflator:



Finally, the exports-imports ratio at current prices is a little skewed.



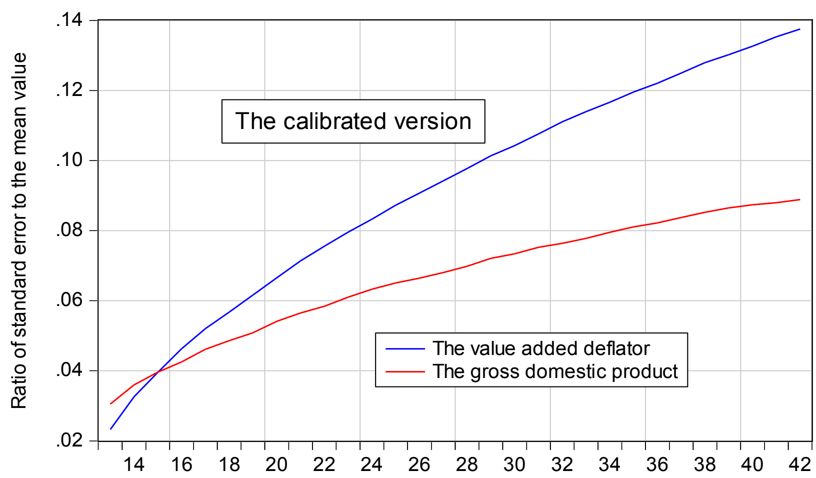
3.5.5.6 Including the Error on Coefficients

Let us now add to the picture the error on coefficients.

This calls for a change in the equations. Considering the estimation results, we see that some of the error correcting coefficients are not significant, in particular for the value added deflator and the wage rate. This means that a random draw will give them the wrong sign in some cases, which will make the model diverge[32]. This calls for a calibration, just as we did for the analysis of shocks.

We shall set both coefficients to 0.10.

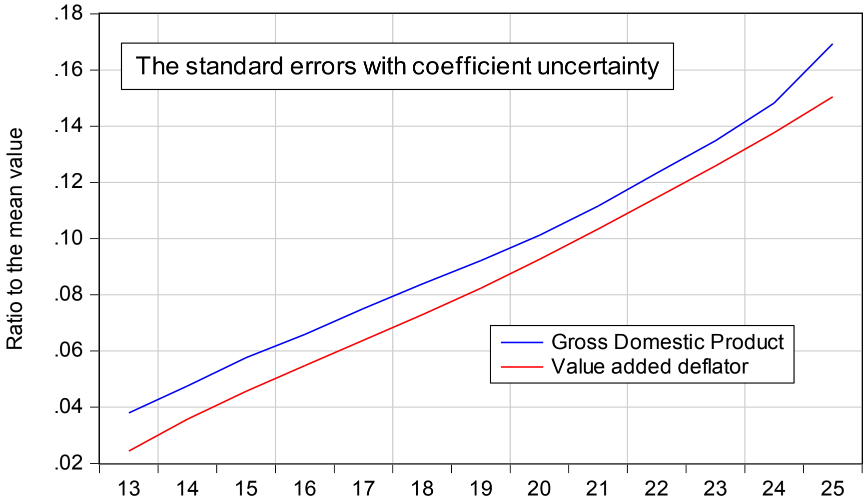
First, let us consider the consequences for a stochastic simulation without coefficient uncertainty.

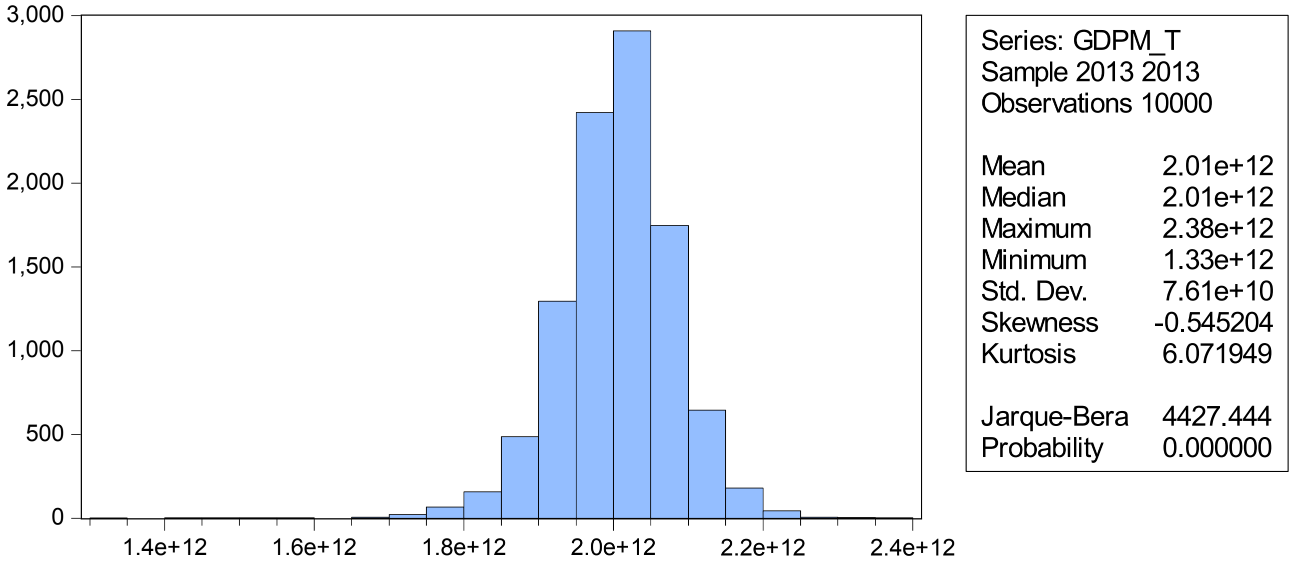


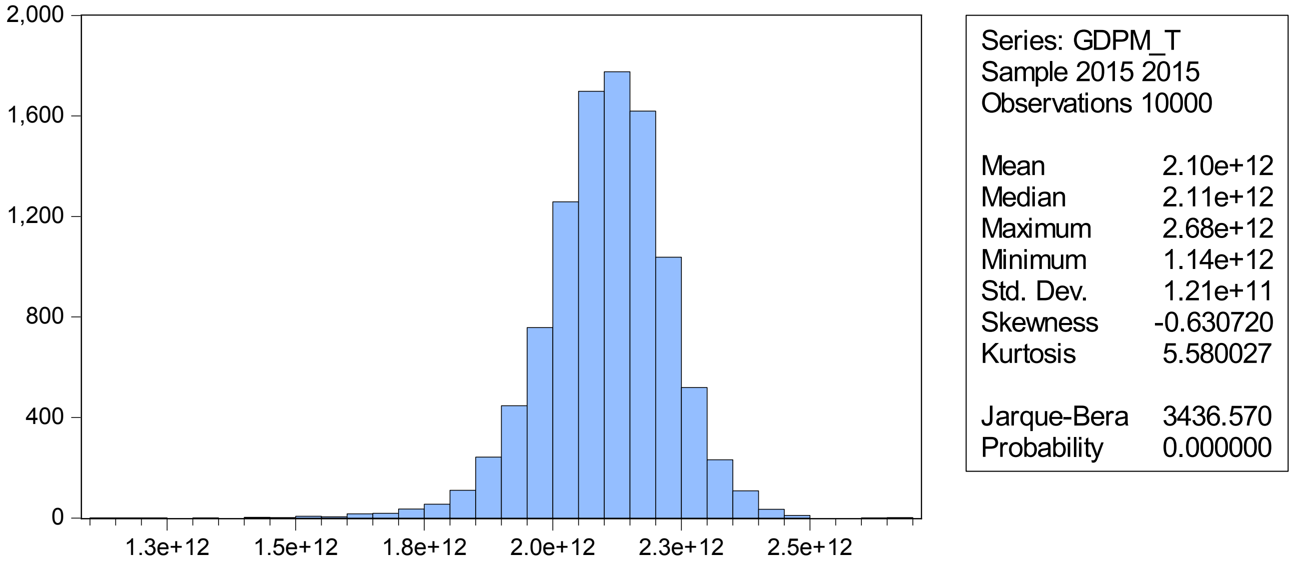
We see that the results do not change much. However, as could be expected, the profile of the price error shows that convergence to a stable value will happen sooner.

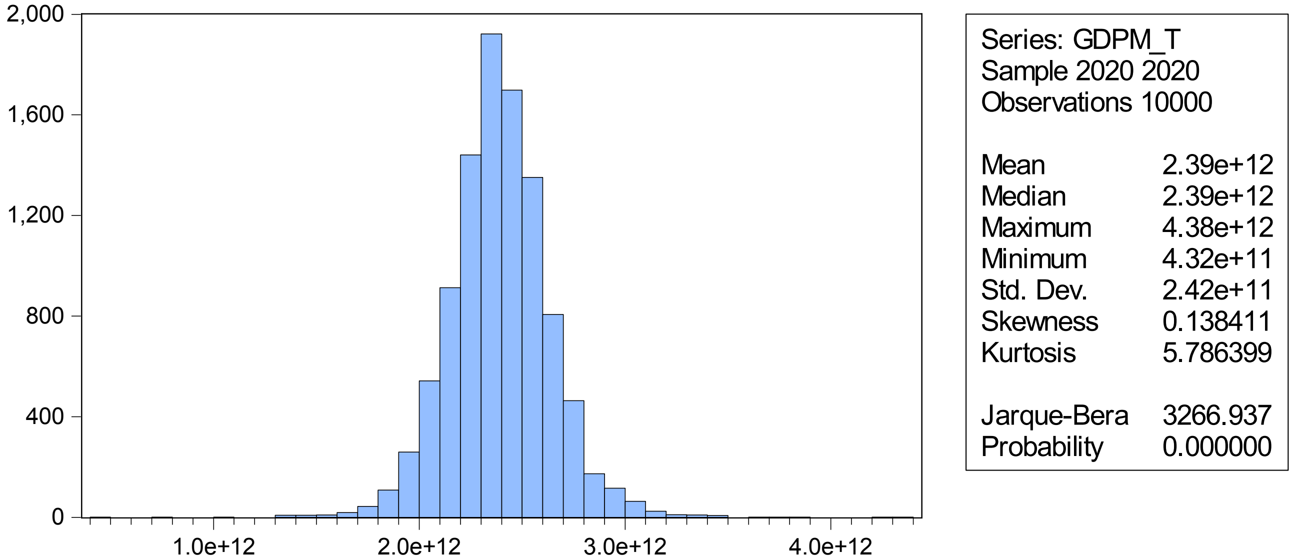
[32] As the error correction process will actually add to the previous error.

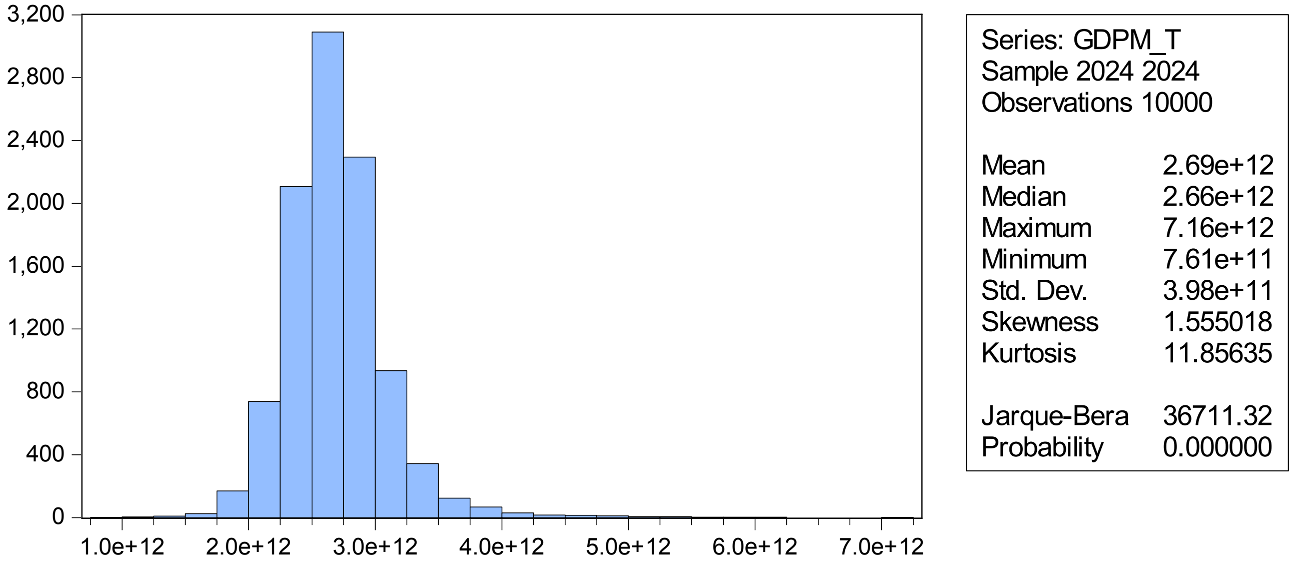
Let us now introduce the error on coefficients. This looks quite risky, as many of them have quite imprecise values (including sometimes their sign). Of course, we have calibrated the most dangerous of them, but this might not be enough.
Indeed, to make EViews succeed in the process, we have to restrict the horizon to 2025.






Series: GDPM_T
Sample 2015 2015
Observations 10000
Mean 2.10e+12
Median 2.11e+12
Maximum 2.68e+12
Minimum 1.14e+12
Std. Dev. 1.21e+11
Skewness -0.630720
Kurtosis 5.580027
Jarque-Bera 3436.570
Probability 0.000000
2,000
1,600
1,200
800
400
0
1.3e+12
1.5e+12
1.8e+12
2.0e+12
2.3e+12
2.5e+12


Series: GDPM_T
Sample 2020 2020
Observations 10000
Mean 2.39e+12
Median 2.39e+12
Maximum 4.38e+12
Minimum 4.32e+11
Std. Dev. 2.42e+11
Skewness 0.138411
Kurtosis 5.786399
Jarque-Bera 3266.937
Probability 0.000000
2,000
1,600
1,200
800
400
0
1.0e+12
2.0e+12
3.0e+12
4.0e+12


Series: GDPM_T
Sample 2024 2024
Observations 10000
Mean 2.69e+12
Median 2.66e+12
Maximum 7.16e+12
Minimum 7.61e+11
Std. Dev. 3.98e+11
Skewness 1.555018
Kurtosis 11.85635
Jarque-Bera 36711.32
Probability 0.000000
3,200
2,800
2,400
2,000
1,600
1,200
800
400
0
1.0e+12
2.0e+12
3.0e+12
4.0e+12
5.0e+12
6.0e+12
7.0e+12

3.6 Rational Expectations

EViews refers to them as "model consistent expectations," and their rationale is a little different from the usual one.

3.6.1 The Framework

In a rational expectations framework, we suppose that some of the agents (at least) are aware of the future assumptions, and are able to evaluate their influence on some (at least) of the model variables. This does not need the knowledge of the actual model equations, just the mathematical « application » from the assumptions to the endogenous.

As they will use this knowledge in their present decisions, some of the present variables will depend on future values, either of assumptions or endogenous elements depending on these assumptions.

To take into account rational expectations one does not need to believe in them. Interpreting the differences in economic behavior (and their consequences for the equilibrium) between forward and backward looking agents is quite interesting if only from a theoretical point of view. The following example will shed some light on this point.

3.6.2 Consequences for Model Simulations

In this context, we can no longer compute the solution for each period separately, moving from the past to the future. The solution for a given (future period) will depend on values for which the solution has not been yet obtained.

This introduces a problem, namely finding a way to take into account future values belonging to the forecasting period. The most popular options are:

- The Fair-Taylor algorithm, which solves the periods in the usual way, then goes back to the first one and iterates (in a Gauss – Seidel way) until convergence of the whole system.
- The Laffargue (1990) algorithm, which basically adds a time dimension to the model equations (duplicating them as many times as there are periods) and solves the associated model as a whole, using the fact that the matrix is band diagonal.

3.6.3 Our Example

In our example, we shall consider two forward influences:

- An adaptation of productive investment to future growth and output gap.
- An adaptation of household consumption to future revenue.

Let us first consider investment. The firms can be able to forecast two elements:

  - Future growth to which they have to adapt their productive capacity.
  - The output gap which they have to close.

The first point is clear, but of course one must be aware that forecasting growth will have a strong impact on growth itself, through both investment and the adaptation of employment to the new growth (we shall suppose employment itself is adapting fast enough for future values to have no influence on decisions).

The second is less, as firms might be tempted to adapt permanently the rate of use permanently to the target.

3.6.4 The Tests

We shall consider the following models:

3.6.4.1 Investment

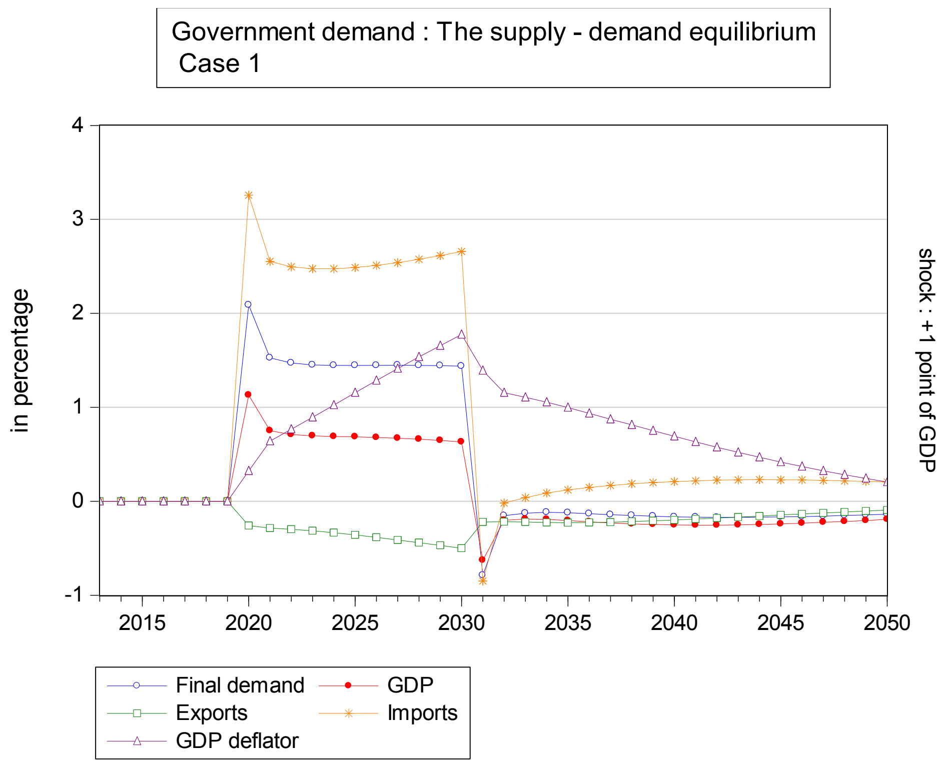
The firms adapt investment to the growth rate of value added. It can be:

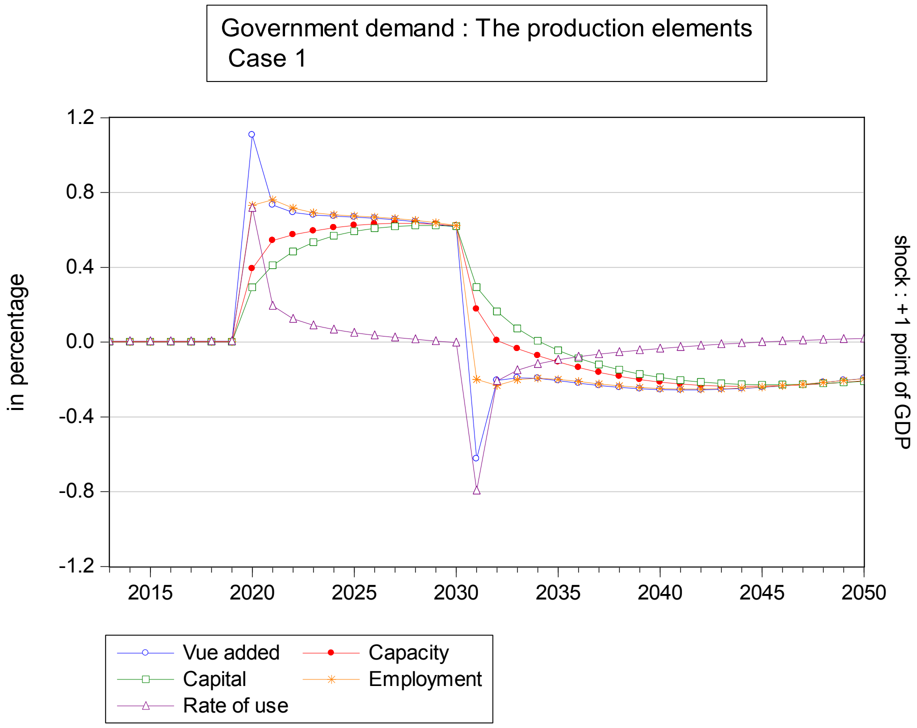
- The present annual growth rate (actually a backward looking formulation).
- The future annual growth rate.
- The average growth rate 8 periods ahead (the average of the next 8 growth rates).

We shall also suppose that firms can forecast the rate of use, and take it into account in their behavior.

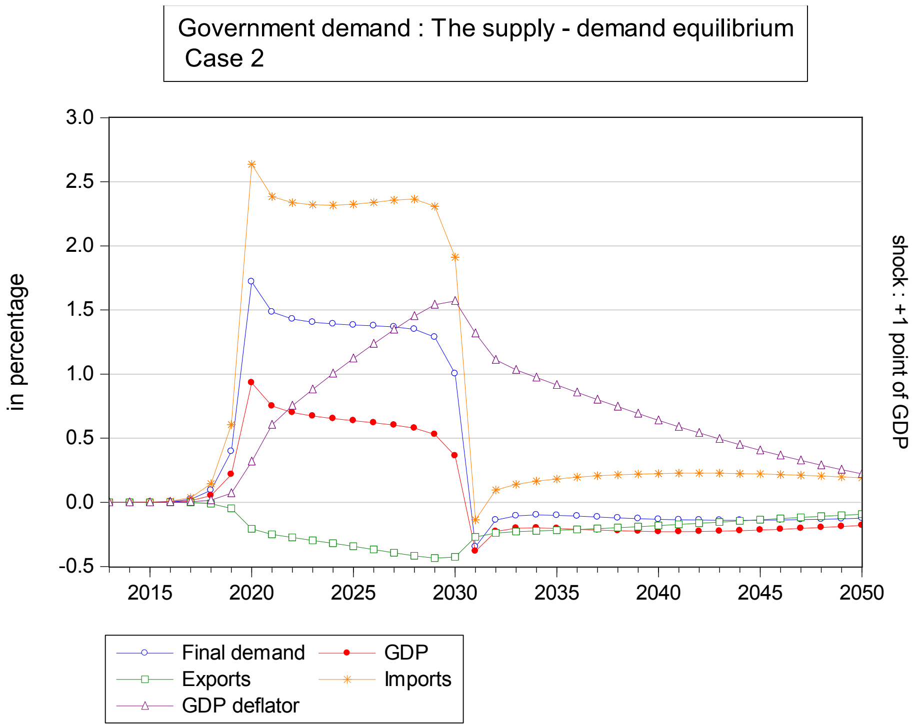
We shall also consider that the state increases its demand by 1% of GDP (as in the previous shocks), but only from 2020 to 2030.

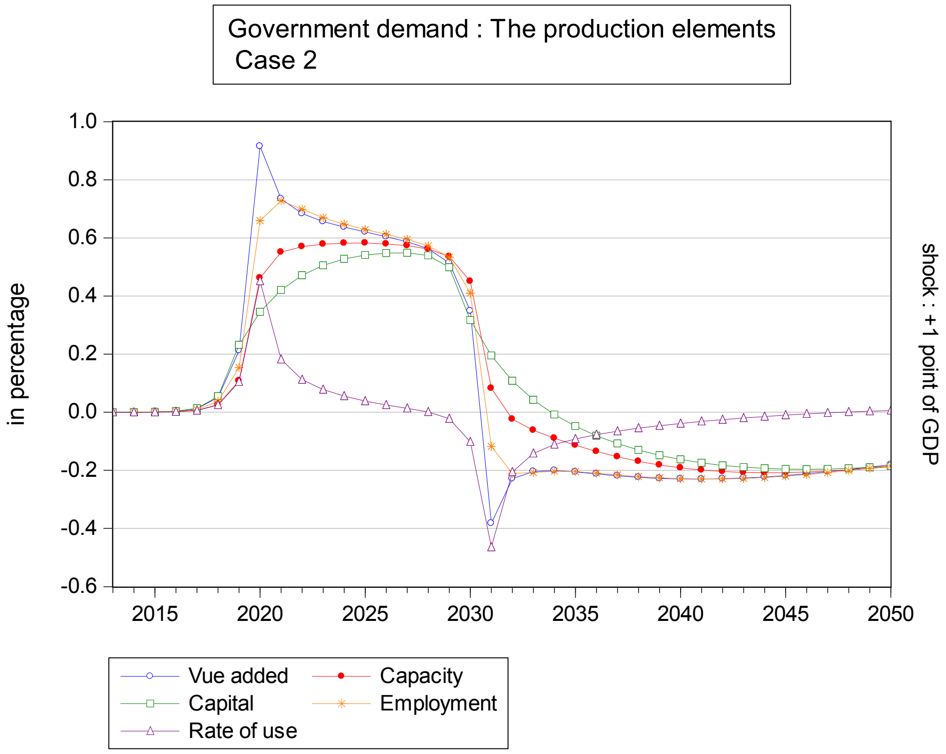
In Case 1, we see the usual consequences of a demand shock using a backward looking model. The reaction to both decisions is symmetrical.





In Case 2, the firms are aware of the change in policy. They start reacting earlier; and, as they are aware that the shock is not permanent, their reaction is a little lower. The rate of use goes back to normal faster.





In case 3, they are even more aware of future evolutions. They react faster (the rate of use decreases in the beginning).

The evolution of capacity elements is much lower. Remember also that a lower growth and rate of use brings less investment and employment, bringing growth even lower.

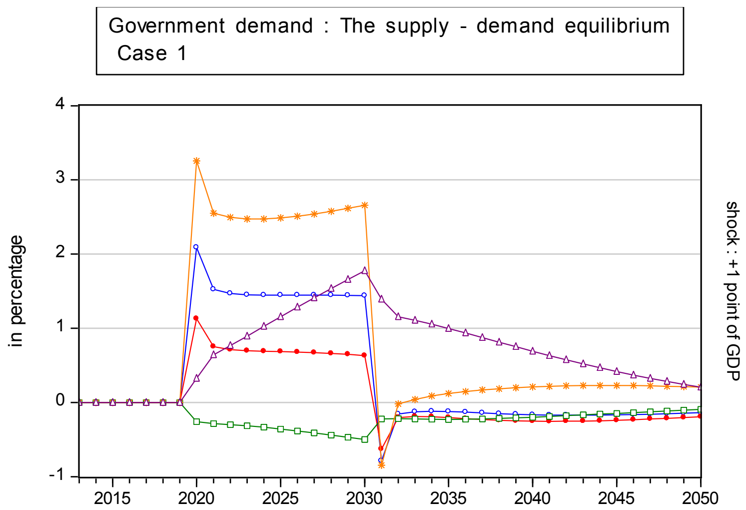
Actually if forecasting the future releases the tensions somewhat, it rather reduces activity in the case of a future negative shock. Making firms believe that the shock is permanent us actually good for the economy, but we do not take into account the negative impact of having to maintain unused capacities.

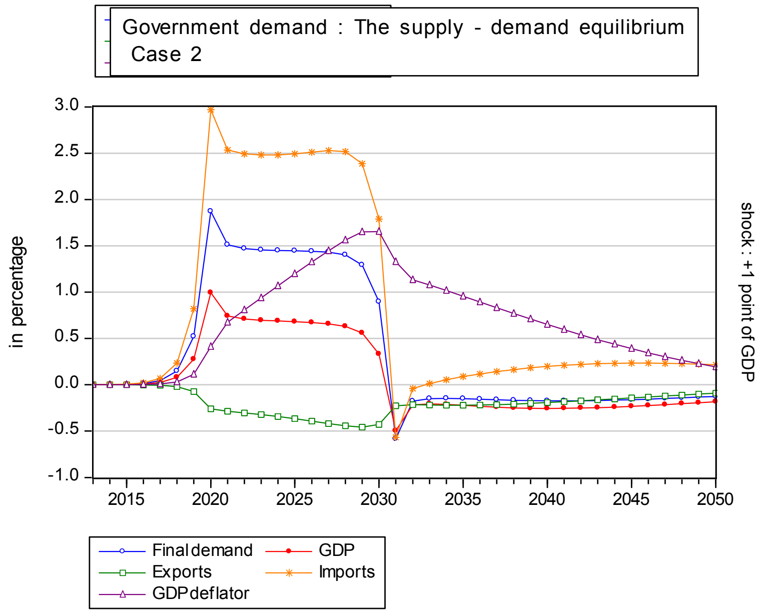
The changes appear sooner before the shock than the forward horizon itself. The early response of forward looking firms is taken into account even earlier by other firms, introducing a cascading effect (rather limited, however).

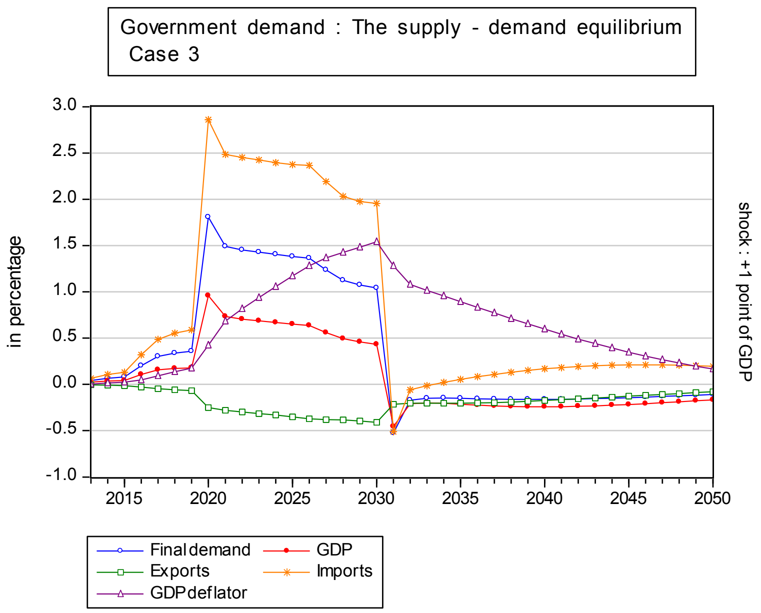
3.6.4.2 Consumption

We will suppose that households can:

- Take only into account present and past revenues.
- For revenue, consider the value of the next year.
- Consider the revenue in four years.


Government demand : The supply - demand equilibrium
Case 1
in percentage
shock : +1 point of GDP
4
3
2
1
0
-1
2015
2020
2025
2030
2035
2040
2045
2050

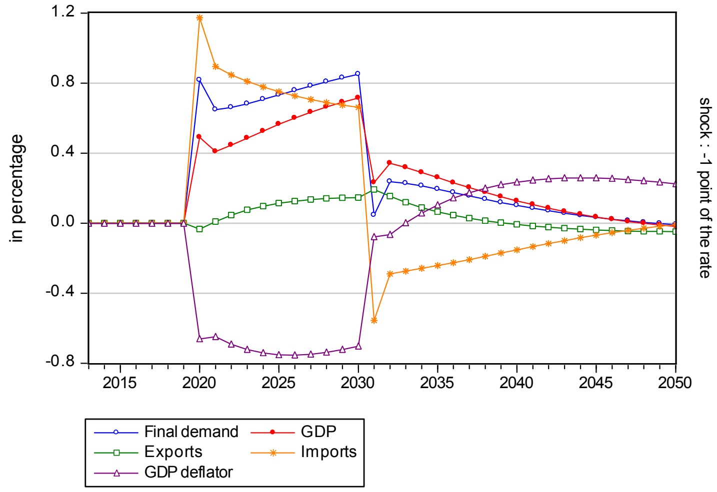

Government demand : The supply - demand equilibrium
Case 2
in percentage
shock : +1 point of GDP
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
2015
2020
2025
2030
2035
2040
2045
2050
Final demand
GDP
Exports
Imports
GDP deflator


Government demand : The supply - demand equilibrium
Case 3
in percentage
shock : +1 point of GDP
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
2015
2020
2025
2030
2035
2040
2045
2050
Final demand
GDP
Exports
Imports
GDP deflator

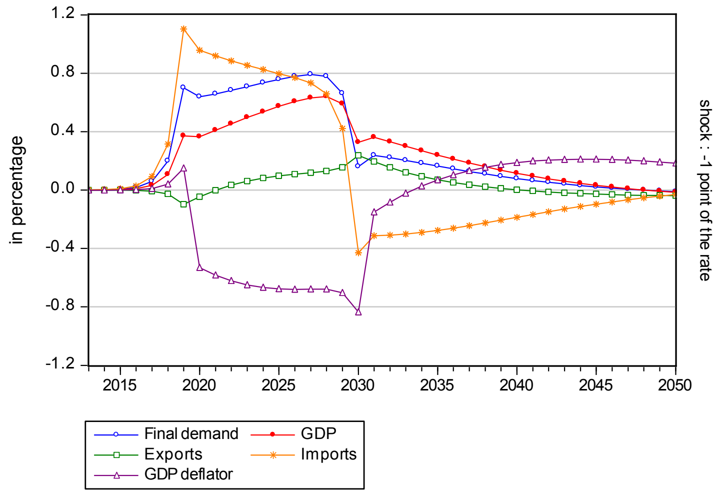
The results are consistent with the previous cases. Forecasting one year ahead smoothes the evolutions, and a longer horizon reduces the impact of the shock.

A more original situation is the case of a decrease in the VAT rate, by one point here, where forecasting four periods ahead makes the consumption increase and decrease appear sooner and with smoother dynamics.

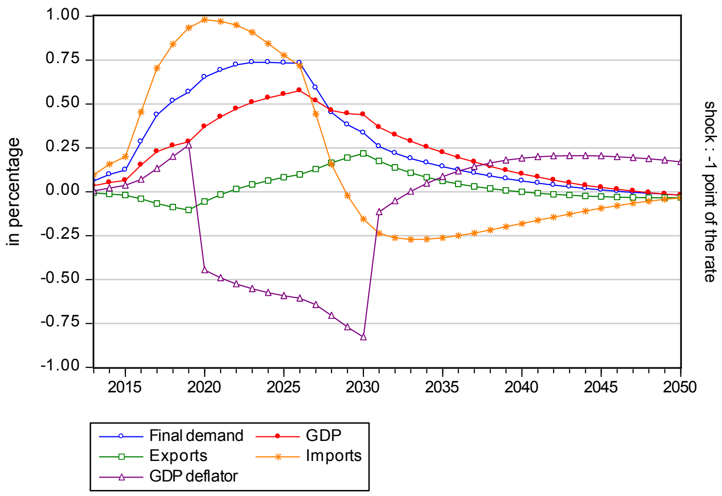
the Value added tax : The supply - demand equilibrium
Case 1



the Value added tax : The supply - demand equilibrium
Case 2



the Value added tax : The supply - demand equilibrium
Case 3



4 Another Country: Sénégal

To show that the South African case is not unique, we shall now apply the same principles to another country: Senegal.

However, if we will manage to obtain a working model, this asked for a lot more compromises than the first case. We have to consider, however, that

- We are not a specialist of the economy of Senegal,
- The time allowed to work on the subject was limited,
- And probably the most important, we are not aware of the data resources available in Senegal, a task that a local researcher, or an economist familiar with the subject, would have done much better. So this work should only be considered as a first step, and we welcome anybody interested to pursue it, perhaps using the tools we have developed, and which are provided with this document.

4.1 Reading the Data: Sen_Read.Prg

We have used the same methodology as for South Africa, starting from the same World Bank file. However, we have not been able to complement it by ILO data, as this organization provides only one observation.

4.2 Generating the Model Series: Sen_Genr.Prg

Again, we used the same method, the only differences coming from missing elements and periods.

4.3 Building the Model Framework and Behaviors: Sen_Model.Prg

We have used the same approach as for South Africa. However, econometric failures have been more numerous, and we had to calibrate about half of the behavioral equations: the four price equations and the employment/unemployment.

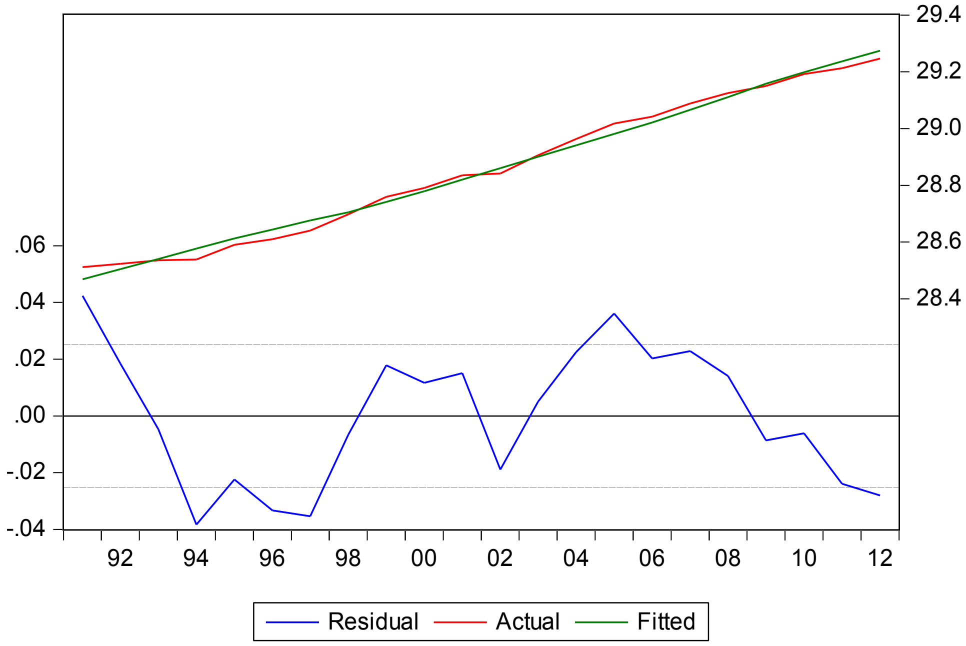
4.3.1 Capacity

The coefficient separating the roles of employment and capital is acceptable.

Dependent Variable: LOG(CAP)
Method: Least Squares
Date: 08/06/14 Time: 02:32
Sample: 1991 2012
Included observations: 22
LOG(CAP)=(1-C_CAP(1))*LOG(K(-1))+C_CAP(1)*LOG(LF)+C_CAP(3)
+C_CAP(4)*(T-2012)*(T<=2012)+C_CAP(1)*LOG((1+TXQ)/(1
+TXN))*(T-2012)*(T>=2012)

	Coefficient	Std. Error	t-Statistic	Prob.
C_CAP(1)	0.533606	0.224543	2.376409	0.0282
C_CAP(3)	7.066538	3.256163	2.170204	0.0429
C_CAP(4)	0.003167	0.002538	1.247985	0.2272

R-squared	0.990974	Mean dependent var	28.85545
Adjusted R-squared	0.990024	S.D. dependent var	0.251669
S.E. of regression	0.025136	Akaike info criterion	-4.402890
Sum squared resid	0.012005	Schwarz criterion	-4.254112
Log likelihood	51.43179	Hannan-Quinn criter.	-4.367843
F-statistic	1043.067	Durbin-Watson stat	0.624582
Prob(F-statistic)	0.000000		



4.3.2 Productive Investment

The coefficients are barely acceptable, with reasonable values

Dependent Variable: IP/K(-1)
Method: Least Squares
Date: 08/06/14 Time: 02:32
Sample: 1996 2012
Included observations: 17
IP/K(-1)=C_IP(1)*IP(-1)/K(-2)+C_IP(2)*@PCH(Q)+C_IP(2)*LOG(UR)
+C_IP(4)+IP_EC

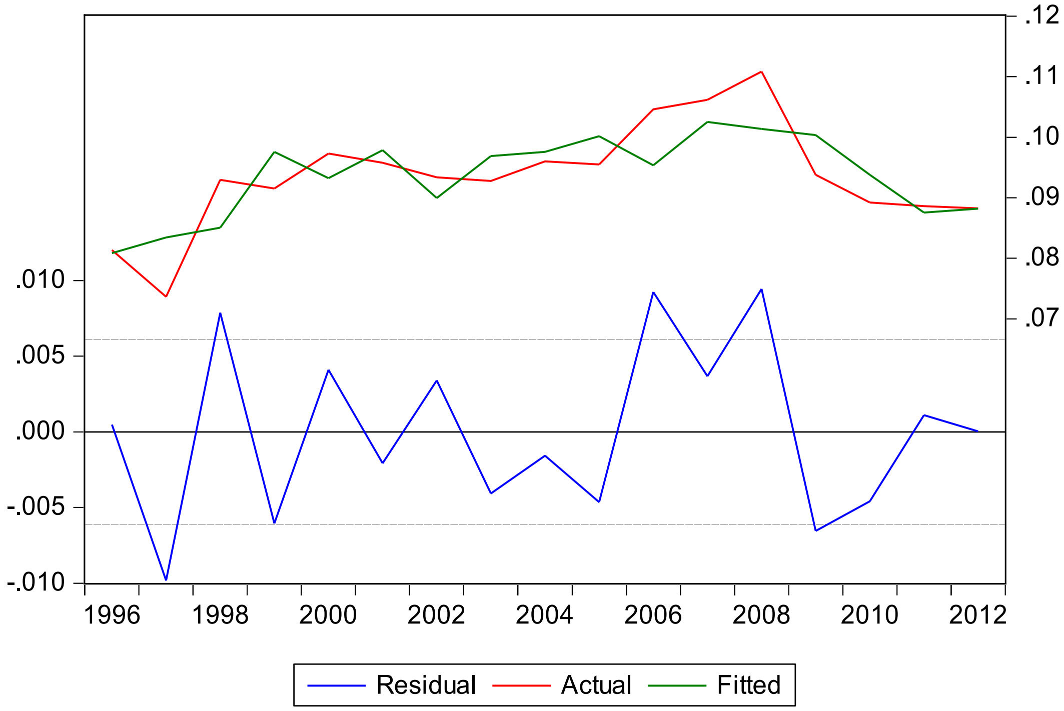
	Coefficient	Std. Error	t-Statistic	Prob.
C_IP(1)	0.504497	0.168237	2.998723	0.0096
C_IP(2)	0.096802	0.045265	2.138557	0.0506
C_IP(4)	0.042851	0.015285	2.803496	0.0141

R-squared	0.576348	Mean dependent var	0.093626
Adjusted R-squared	0.515827	S.D. dependent var	0.008772
S.E. of regression	0.006104	Akaike info criterion	-7.201040
Sum squared resid	0.000522	Schwarz criterion	-7.054003
Log likelihood	64.20884	Hannan-Quinn criter.	-7.186424
F-statistic	9.523013	Durbin-Watson stat	2.695387
Prob(F-statistic)	0.002449		

Dependent Variable: IP/K(-1)
Method: Least Squares
Date: 08/06/14 Time: 02:32
Sample: 1996 2012
Included observations: 17
IP/K(-1)=C_IP(1)*IP(-1)/K(-2)+C_IP(2)*@PCH(Q)+C_IP(2)*LOG(UR)
+C_IP(4)+IP_EC

	Coefficient	Std. Error	t-Statistic	Prob.
C_IP(1)	0.504497	0.168237	2.998723	0.0096
C_IP(2)	0.096802	0.045265	2.138557	0.0506
C_IP(4)	0.042851	0.015285	2.803496	0.0141

R-squared	0.576348	Mean dependent var	0.093626
Adjusted R-squared	0.515827	S.D. dependent var	0.008772
S.E. of regression	0.006104	Akaike info criterion	-7.201040
Sum squared resid	0.000522	Schwarz criterion	-7.054003
Log likelihood	64.20884	Hannan-Quinn criter.	-7.186424
F-statistic	9.523013	Durbin-Watson stat	2.695387
Prob(F-statistic)	0.002449		

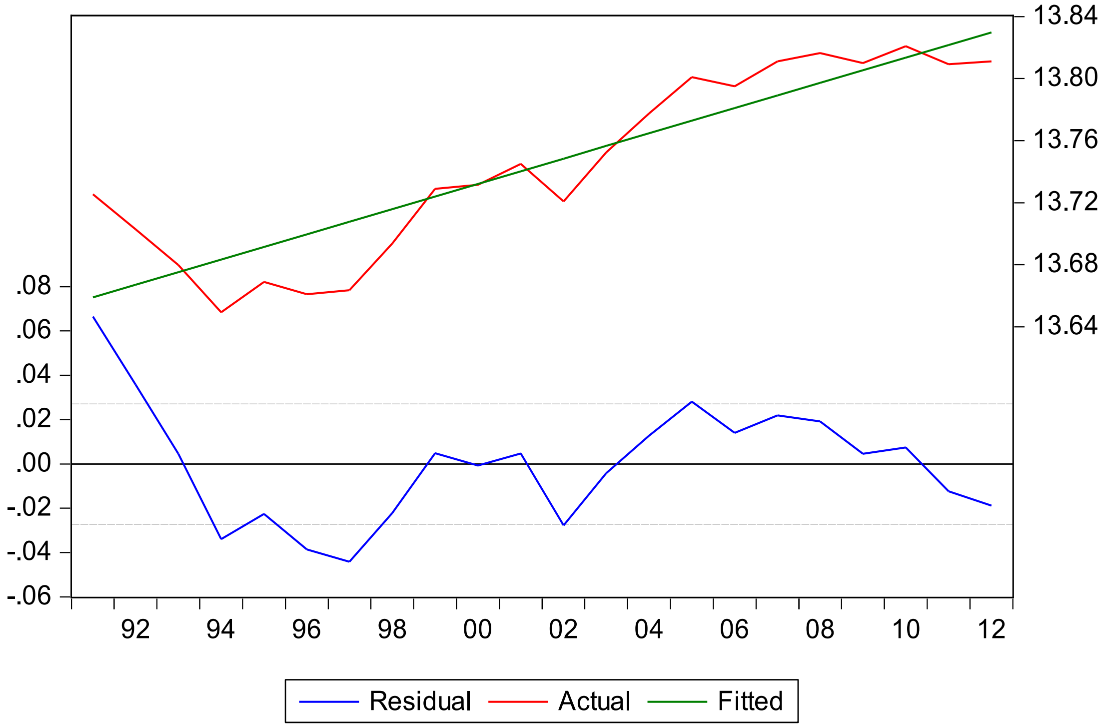


4.3.3 Labor Productivity Trend

Dependent Variable: LOG(LPT)
Method: Least Squares
Date: 08/06/14 Time: 02:32
Sample (adjusted): 1991 2012
Included observations: 22 after adjustments
LOG(LPT)=C_LPT(1)+C_LPT(2)*(T-2012)*(T<=2012)+LOG((1+TXQ)/(1
+TXN))*(T-2012)*(T>=2012)

	Coefficient	Std. Error	t-Statistic	Prob.
C_LPT(1)	13.82981	0.011188	1236.080	0.0000
C_LPT(2)	0.008137	0.000912	8.922139	0.0000

R-squared	0.799206	Mean dependent var	13.74437
Adjusted R-squared	0.789166	S.D. dependent var	0.059105
S.E. of regression	0.027139	Akaike info criterion	-4.289173
Sum squared resid	0.014731	Schwarz criterion	-4.189988
Log likelihood	49.18091	Hannan-Quinn criter.	-4.265808
F-statistic	79.60457	Durbin-Watson stat	0.553002
Prob(F-statistic)	0.000000		



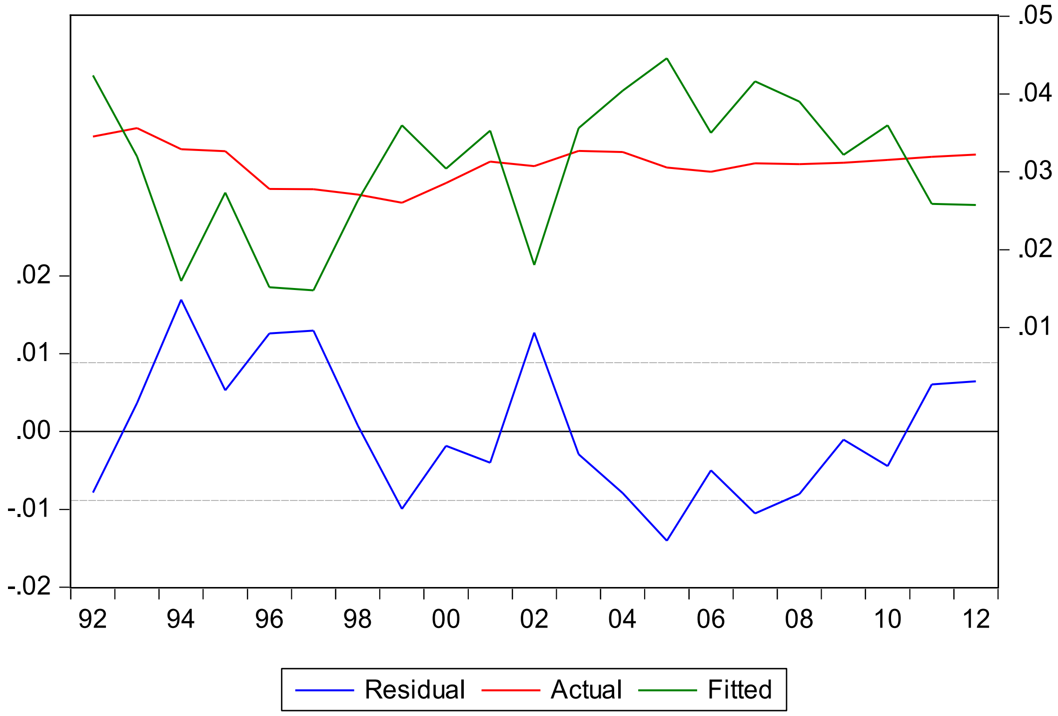
4.3.4 Employment

We had to calibrate the equation

Dependent Variable: DLOG(LF)
Method: Least Squares
Date: 08/06/14 Time: 02:32
Sample (adjusted): 1992 2012
Included observations: 21 after adjustments
DLOG(LF)=0.5*DLOG(LFD)+0.35*LOG(LFD(-1)/LF(-1))+C_LF(3)+LF_EC

	Coefficient	Std. Error	t-Statistic	Prob.
C_LF(3)	0.017188	0.001931	8.900963	0.0000

R-squared	-12.484335	Mean dependent var	0.030940
Adjusted R-squared	-12.484335	S.D. dependent var	0.002410
S.E. of regression	0.008849	Akaike info criterion	-6.570506
Sum squared resid	0.001566	Schwarz criterion	-6.520767
Log likelihood	69.99032	Hannan-Quinn criter.	-6.559712
Durbin-Watson stat	1.091431		



4.3.5 Unemployment

Again, we used calibration.

Dependent Variable: D(POPAC)/POP1564(-1)
Method: Least Squares
Date: 08/06/14 Time: 02:32
Sample (adjusted): 1992 2012
Included observations: 21 after adjustments
D(POPAC)/POP1564(-1)=0.3*D(LT)/POP1564(-1)+0.2*D(POP1564)
/POP1564(-1)-0.2*(POPAC(-1)/POP1564(-1)-0.3*LT(-1)/POP1564(
-1)-C_POPAC(5))+0*(T-2012)*(T<=2012)+POPAC_EC

	Coefficient	Std. Error	t-Statistic	Prob.
C_POPAC(5)	0.614934	0.001293	475.7566	0.0000

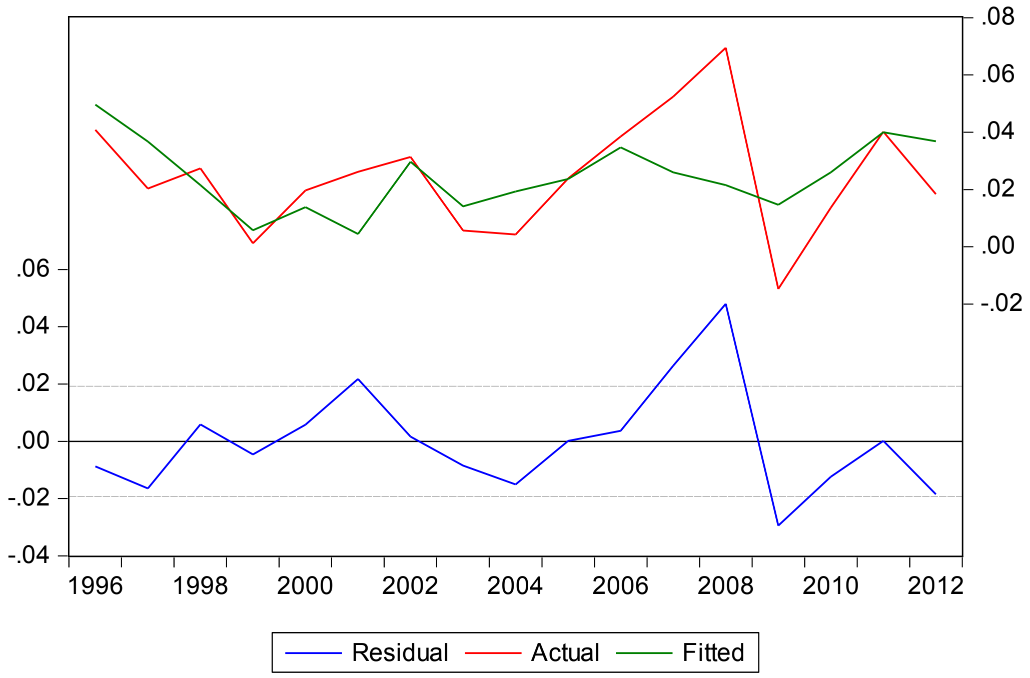
R-squared	0.611095	Mean dependent var	0.023971
Adjusted R-squared	0.611095	S.D. dependent var	0.001900
S.E. of regression	0.001185	Akaike info criterion	-10.59232
Sum squared resid	2.81E-05	Schwarz criterion	-10.54258
Log likelihood	112.2194	Hannan-Quinn criter.	-10.58153
Durbin-Watson stat	0.455924		

Value Added Deflator

Dependent Variable: DLOG(PQ)
Method: Least Squares
Date: 08/06/14 Time: 02:51
Sample: 1996 2012
Included observations: 17
DLOG(PQ)=0.6*DLOG(UWC)+0.2*DLOG(UR)-0.3*(LOG(PQ(-1)/UWC(
-1))-0.2*LOG(UR(-1)))+C_PQ(5)+C_PQ(6)*(T-2012)*(T<=2012)

	Coefficient	**Std. Error**	**t-Statistic**	**Prob.**
C_PQ(5)	0.752625	0.008958	84.01398	0.0000
C_PQ(6)	0.012001	0.000955	12.56709	0.0000

R-squared	0.155251	Mean dependent var	0.024625
Adjusted R-squared	0.098935	S.D. dependent var	0.020321
S.E. of regression	0.019289	Akaike info criterion	-4.948403
Sum squared resid	0.005581	Schwarz criterion	-4.850378
Log likelihood	44.06143	Hannan-Quinn criter.	-4.938659
F-statistic	2.756763	Durbin-Watson stat	1.716136
Prob(F-statistic)	0.117598		

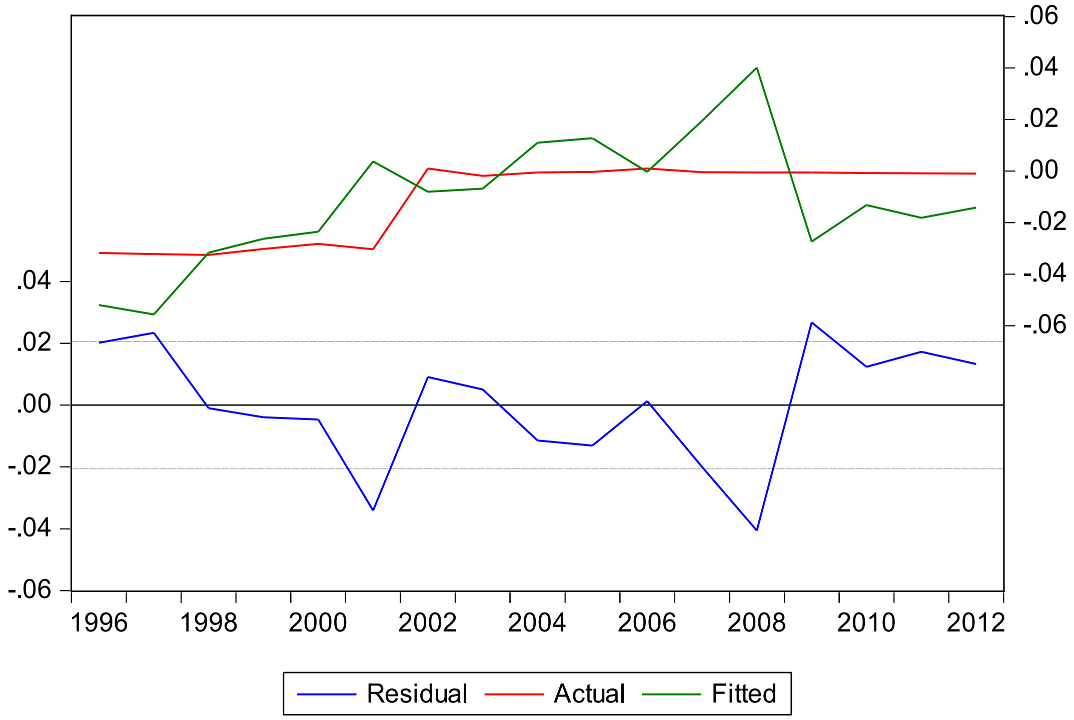


4.3.6 Wage Rate

Dependent Variable: DLOG(WR)
Method: Least Squares
Date: 08/06/14 Time: 02:32
Sample: 1996 2012
Included observations: 17
DLOG(WR)=0.6*DLOG(PCOH)+0.7*DLOG(LP)-0.01*LOG(UNR)-0.35
*(LOG(UWC(-1))-0.5*LOG(PCOH(-1))-0.5*LOG(PQ(-1)))+C_WR(4)
+C_WR(5)*(T-2012)*(T<=2012)+C_WR(6)*(T<2002)

	Coefficient	Std. Error	t-Statistic	Prob.
C_WR(4)	-0.839329	0.011050	-75.95517	0.0000
C_WR(5)	-0.014094	0.001827	-7.715152	0.0000
C_WR(6)	-0.016420	0.018727	-0.876814	0.3954

R-squared	-0.643345	Mean dependent var	-0.011159
Adjusted R-squared	-0.878109	S.D. dependent var	0.015052
S.E. of regression	0.020627	Akaike info criterion	-4.765614
Sum squared resid	0.005957	Schwarz criterion	-4.618576
Log likelihood	43.50772	Hannan-Quinn criter.	-4.750998
Durbin-Watson stat	1.594638		

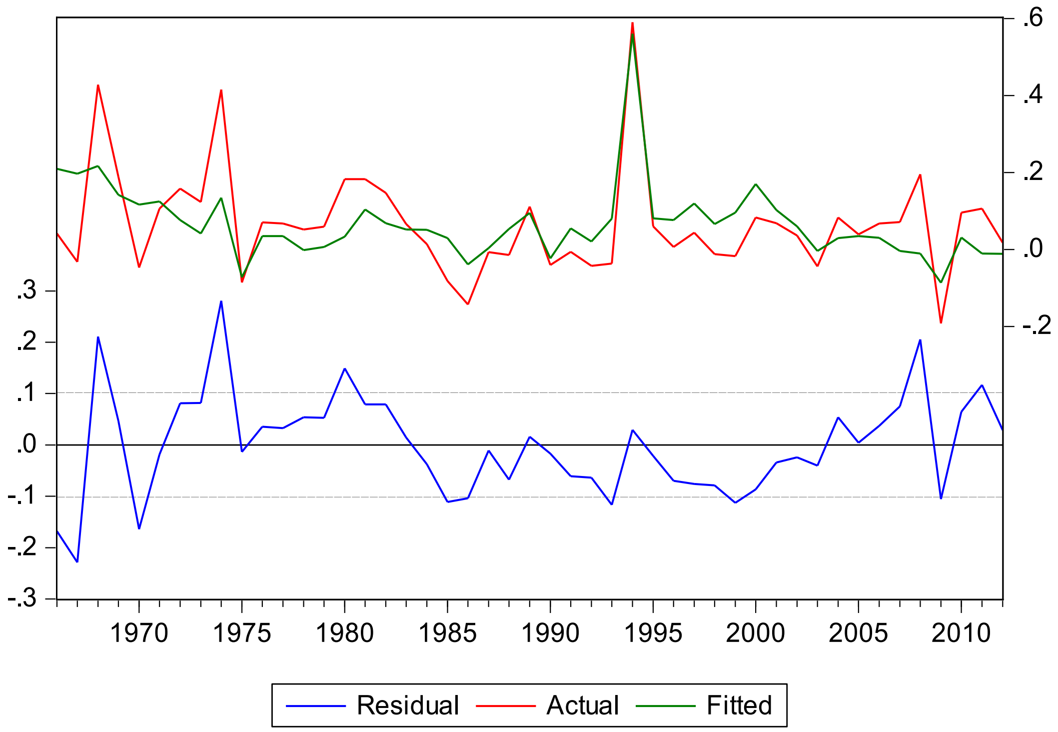


4.3.7 Imports Deflator

Dependent Variable: DLOG(PM)
Method: Least Squares
Date: 08/06/14 Time: 02:32
Sample (adjusted): 1966 2012
Included observations: 47 after adjustments
DLOG(PM)=0.2*DLOG(PP)+0.6*DLOG(PPX*ER)-0.3*(LOG(PM(-1))-0.1
*LOG(PP(-1))-(1-0.1)*LOG(PPX(-1)*ER(-1)))+C_PM(5)*(T-2012)
(T<=2012)(T<=2012)+C_PM(6) +PM_EC

	Coefficient	Std. Error	t-Statistic	Prob.
C_PM(5)	-0.002536	0.001093	-2.320503	0.0249
C_PM(6)	-1.629141	0.029183	-55.82587	0.0000

R-squared	0.469643	Mean dependent var	0.067576
Adjusted R-squared	0.457857	S.D. dependent var	0.138032
S.E. of regression	0.101633	Akaike info criterion	-1.693268
Sum squared resid	0.464821	Schwarz criterion	-1.614538
Log likelihood	41.79180	Hannan-Quinn criter.	-1.663642
F-statistic	39.84853	Durbin-Watson stat	1.460283
Prob(F-statistic)	0.000000		

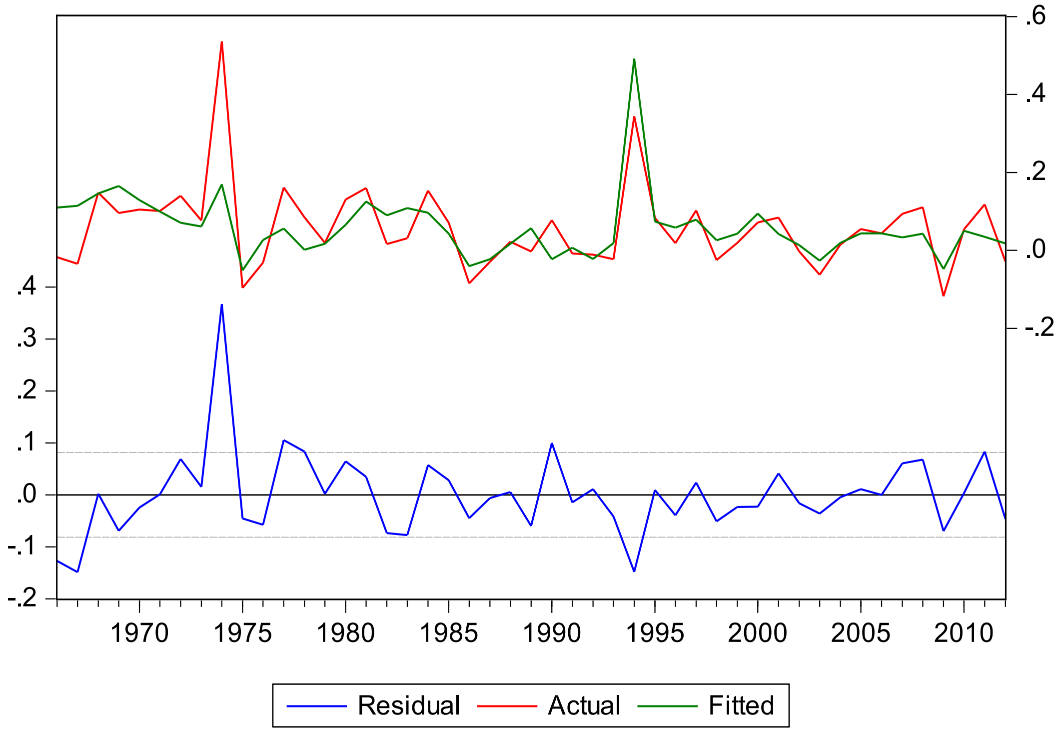


4.3.8 Exports Deflator

Dependent Variable: DLOG(PX)
Method: Least Squares
Date: 08/06/14 Time: 02:32
Sample (adjusted): 1966 2012
Included observations: 47 after adjustments
DLOG(PX)=0.6*DLOG(PP)+0.4*DLOG(PPX*ER)-0.3*(LOG(PX(-1))-0.6
*LOG(PP(-1))-(1-0.6)*LOG(PPX(-1)*ER(-1)))+C_PX(5)*(T-2012)
*(T<=2012)+C_PX(6) +PX_EC

	Coefficient	Std. Error	t-Statistic	Prob.
C_PX(5)	-6.61E-05	0.000876	-0.075471	0.9402
C_PX(6)	-0.722881	0.023401	-30.89144	0.0000

R-squared	0.453296	Mean dependent var	0.059038
Adjusted R-squared	0.441147	S.D. dependent var	0.109017
S.E. of regression	0.081497	Akaike info criterion	-2.134879
Sum squared resid	0.298880	Schwarz criterion	-2.056149
Log likelihood	52.16965	Hannan-Quinn criter.	-2.105252
F-statistic	37.31138	Durbin-Watson stat	1.887528
Prob(F-statistic)	0.000000		



.6
.4
.2
.0
-.2
.4
.3
.2
.1
.0
-.1
-.2
1970
1975
1980
1985
1990
1995
2000
2005
2010
Residual
Actual
Fitted

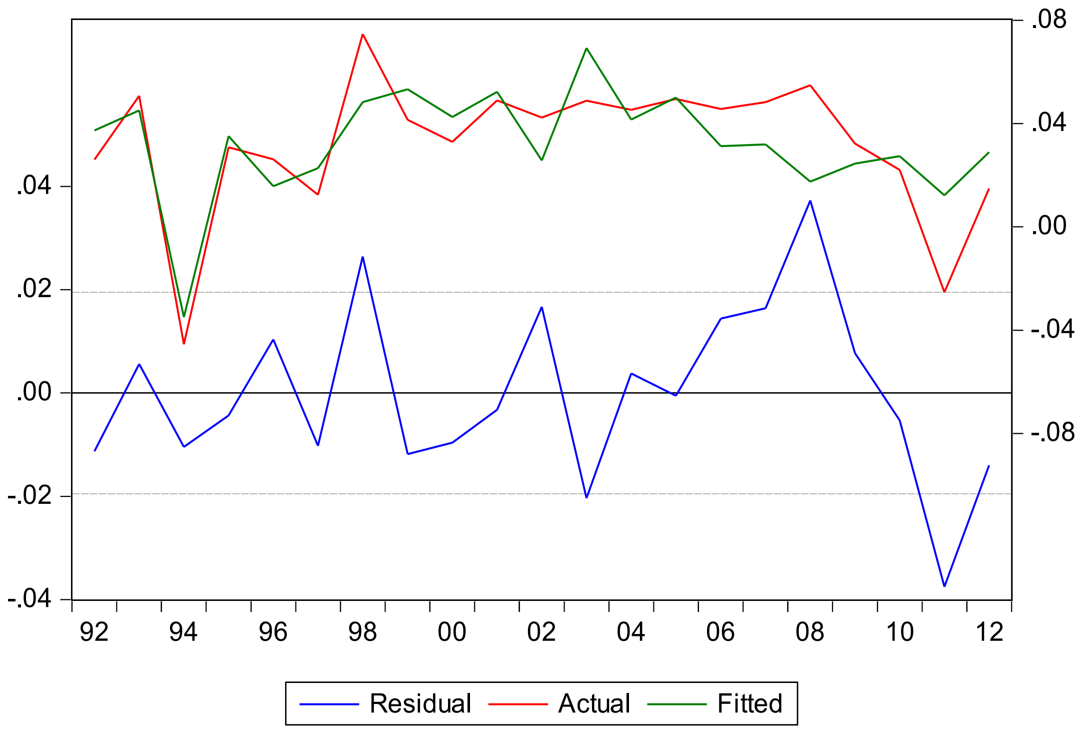
4.3.9 Household Consumption

One of the few cases where we found acceptable coefficients was in the end not significant.

Dependent Variable: DLOG(COH)
Method: Least Squares
Date: 08/06/14 Time: 02:32
Sample (adjusted): 1992 2012
Included observations: 21 after adjustments
DLOG(COH)=C_COH(1)+C_COH(2)*(T-2012)*(T<=2012)+C_COH(3)
*DLOG(HRDI)+C_COH(4)*DLOG(PCOH)+C_COH(5)*LOG(COH(-1)
/HRDI(-1))-0.003*D(UNR)+C_COH(7)*(IRS-100*@PCH(PCOH))
+COH_EC

	Coefficient	Std. Error	t-Statistic	Prob.
C_COH(1)	-0.051198	0.039089	-1.309765	0.2100
C_COH(2)	-0.005474	0.002984	-1.834468	0.0865
C_COH(3)	0.655741	0.224975	2.914736	0.0107
C_COH(4)	-0.317506	0.722515	-0.439446	0.6666
C_COH(5)	-0.455196	0.269994	-1.685945	0.1125
C_COH(7)	-0.002073	0.005984	-0.346477	0.7338

R-squared	0.607296	Mean dependent var	0.032272
Adjusted R-squared	0.476395	S.D. dependent var	0.026956
S.E. of regression	0.019505	Akaike info criterion	-4.801319
Sum squared resid	0.005707	Schwarz criterion	-4.502884
Log likelihood	56.41385	Hannan-Quinn criter.	-4.736551
F-statistic	4.639349	Durbin-Watson stat	1.707797
Prob(F-statistic)	0.009288		



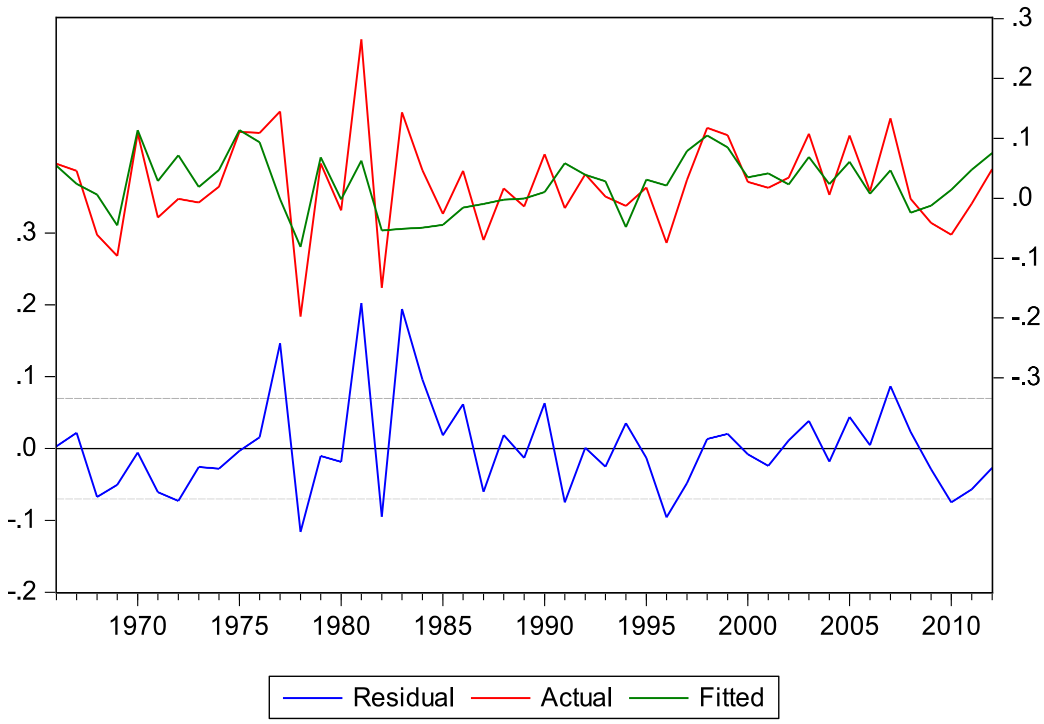
4.3.10 Imports At Constant Prices

Some elements are rather satisfactory.

Dependent Variable: DLOG(M)
Method: Least Squares
Date: 08/06/14 Time: 02:32
Sample (adjusted): 1966 2012
Included observations: 47 after adjustments
Convergence achieved after 9 iterations
DLOG(M)=C_M(1)*DLOG(TD)+0.1*DLOG(UR)+C_M(3)*DLOG(COMPM)
+C_M(4)+C_M(5)*(LOG(M(-1)/TD(-1))-0.1*LOG(UR(-1))-C_M(3)
LOG(COMPM(-1))) +C_M(6)(T-2012)*(T<=2012)+M_EC

	Coefficient	Std. Error	t-Statistic	Prob.
C_M(1)	0.800377	0.295943	2.704495	0.0098
C_M(3)	-0.082799	0.100697	-0.822258	0.4156
C_M(4)	-0.852050	0.238753	-3.568759	0.0009
C_M(5)	-0.479745	0.133794	-3.585706	0.0009
C_M(6)	-0.001132	0.000876	-1.292747	0.2032

R-squared	0.323854	Mean dependent var	0.024667
Adjusted R-squared	0.259459	S.D. dependent var	0.081365
S.E. of regression	0.070019	Akaike info criterion	-2.379821
Sum squared resid	0.205910	Schwarz criterion	-2.182997
Log likelihood	60.92580	Hannan-Quinn criter.	-2.305755
F-statistic	5.029192	Durbin-Watson stat	2.207784
Prob(F-statistic)	0.002104		



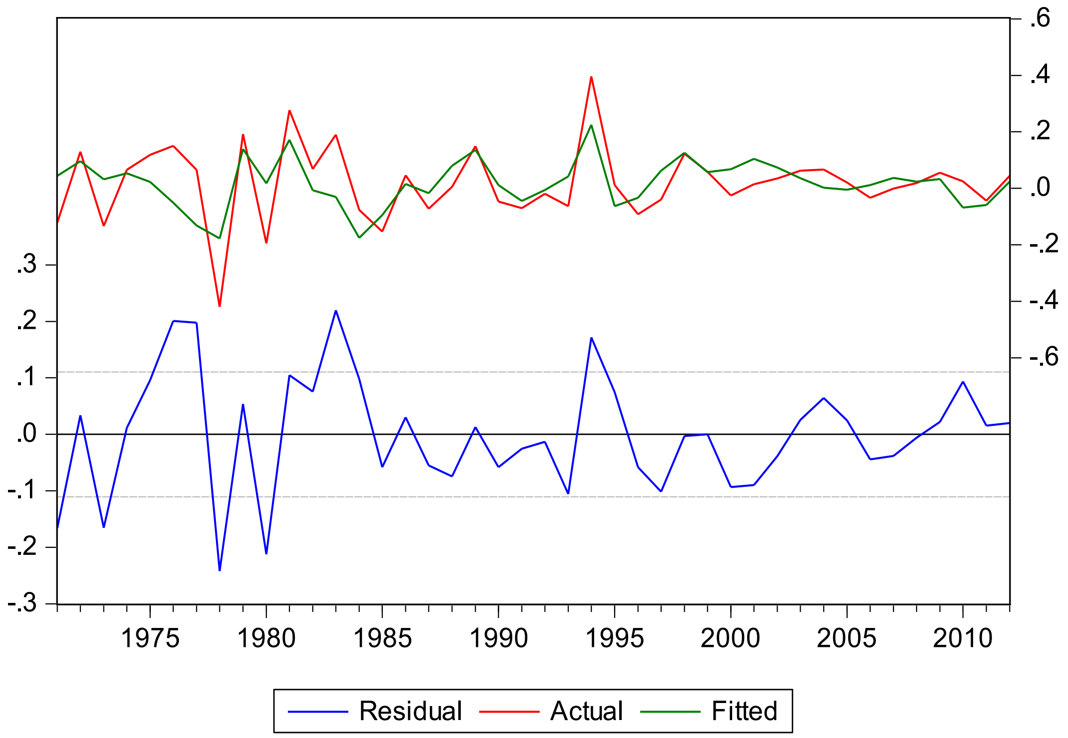
4.3.11 Exports At Constant Prices

The results are not too bad, with a low value of the short term elasticity to world demand.

Dependent Variable: DLOG(X)
Method: Least Squares
Date: 08/06/14 Time: 02:32
Sample (adjusted): 1971 2012
Included observations: 42 after adjustments
DLOG(X)=C_X(1)*DLOG(WD)+C_X(2)*LOG(X(-1)/WD(-1))+C_X(3)
*LOG(UR)+C_X(4)*LOG(COMPX)+C_X(5)+C_X(6)*(T-2012)
*(T<=2012)+X_EC

	Coefficient	Std. Error	t-Statistic	Prob.
C_X(1)	0.157281	0.415276	0.378740	0.7071
C_X(2)	-0.731972	0.158713	-4.611912	0.0000
C_X(3)	-0.193404	0.135249	-1.429986	0.1613
C_X(4)	-0.342164	0.130930	-2.613332	0.0130
C_X(5)	-4.056468	1.022305	-3.967962	0.0003
C_X(6)	-0.024088	0.005474	-4.400302	0.0001

R-squared	0.392616	Mean dependent var	0.018764
Adjusted R-squared	0.308257	S.D. dependent var	0.132887
S.E. of regression	0.110524	Akaike info criterion	-1.435612
Sum squared resid	0.439757	Schwarz criterion	-1.187373
Log likelihood	36.14785	Hannan-Quinn criter.	-1.344622
F-statistic	4.654113	Durbin-Watson stat	1.890231
Prob(F-statistic)	0.002229		



4.4 A Forecast

Using the following theoretical growth rates:

- txq=0.05
- txn=0.01
- txp=0.05

We get the data below.

Table 10. A spontaneous simulation on the future

	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
Real equlibrium										
GDP	8.70	4.25	5.27	5.17	5.19	5.17	5.15	5.13	5.11	5.09
Final demand	9.45	3.74	4.82	4.73	4.79	4.81	4.82	4.83	4.83	4.83
Productive Investment	12.29	2.71	4.61	4.51	4.70	4.79	4.87	4.92	4.96	4.99
Total Investment	10.13	3.36	4.72	4.65	4.78	4.85	4.91	4.95	4.97	4.99
Household Consumption	5.70	4.19	4.69	4.71	4.76	4.76	4.76	4.76	4.75	4.75
Imports	9.03	4.60	5.13	4.91	4.87	4.84	4.81	4.80	4.79	4.79
Exports	5.43	7.44	7.32	6.99	6.66	6.41	6.22	6.07	5.95	5.86
Export-import ratio	67.29	69.12	70.56	71.95	73.18	74.28	75.27	76.18	77.03	77.81
Productive capacity	6.10	5.37	5.22	5.08	5.00	4.94	4.91	4.90	4.89	4.89
Rate of use (*)	99.60	98.56	98.62	98.72	98.92	99.14	99.37	99.59	99.81	100.01

Deflators										
GDP	1.20	0.82	0.85	0.90	1.01	1.15	1.31	1.47	1.63	1.80
Exports	5.43	7.44	7.32	6.99	6.66	6.41	6.22	6.07	5.95	5.86
Imports	1.61	2.45	3.10	3.57	3.91	4.16	4.34	4.47	4.57	4.64
Consumer price index	2.05	1.76	1.78	1.85	1.96	2.09	2.23	2.38	2.52	2.66
Production price	1.63	1.30	1.33	1.39	1.50	1.64	1.79	1.94	2.10	2.25
Wage rate	3.22	2.44	3.61	4.13	4.59	4.94	5.23	5.49	5.72	5.92
Export competitiveness	-6.81	-5.56	-4.53	-3.77	-3.20	-2.77	-2.43	-2.17	-1.96	-1.78
Import competitiveness	-0.02	1.13	1.75	2.15	2.37	2.48	2.51	2.48	2.42	2.34
Employment										
Firms employment	4.01	2.05	1.93	1.65	1.49	1.38	1.30	1.23	1.19	1.15
Total employment	4.01	2.05	1.93	1.65	1.49	1.38	1.30	1.23	1.19	1.15
Unemployment rate (*)	9.57	9.64	9.62	9.66	9.71	9.75	9.78	9.81	9.83	9.84
Others										
Margins rate (*)	91.78	91.79	91.80	91.81	91.81	91.81	91.80	91.80	91.80	91.80
Profits rate (*)	43.47	42.95	42.92	42.83	42.74	42.64	42.52	42.39	42.24	42.09
(*) In growth rates										

The results are rather stable but inflation is quite low (it will converge to the long term value in the end).

If we increase, by 2 points, both the residual on the value added deflator and the wage rate, we get:

Table 11: Improving the spontaneous simulation

	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
Real equlibrium										
GDP	6.56	3.35	4.72	4.87	5.02	5.06	5.07	5.06	5.04	5.03
Final demand	7.11	2.89	4.41	4.63	4.87	4.96	5.01	5.02	5.01	5.00
Productive Investment	8.19	1.35	4.07	4.48	4.88	5.02	5.07	5.07	5.05	5.03
Total Investment	7.24	2.41	4.35	4.64	4.91	5.01	5.05	5.05	5.04	5.02
Household Consumption	3.49	3.08	4.15	4.55	4.81	4.93	4.99	5.01	5.01	5.00
Imports	7.33	3.92	4.80	4.87	4.99	5.04	5.06	5.05	5.04	5.03
Exports	4.89	6.55	6.34	5.99	5.70	5.49	5.35	5.26	5.19	5.15
Export-import ratio	68.00	69.72	70.75	71.50	71.99	72.30	72.50	72.64	72.74	72.83
Productive capacity	5.54	4.77	4.66	4.60	4.60	4.63	4.66	4.69	4.72	4.74
Rate of use (*)	98.16	96.85	96.91	97.17	97.55	97.96	98.34	98.69	99.00	99.28
Deflators										
GDP	5.61	4.99	4.78	4.64	4.56	4.53	4.52	4.52	4.54	4.56
Exports	4.89	6.55	6.34	5.99	5.70	5.49	5.35	5.26	5.19	5.15
Imports	2.39	3.07	3.61	3.99	4.27	4.47	4.61	4.72	4.79	4.85
Consumer price index	5.62	5.08	4.88	4.75	4.68	4.64	4.63	4.63	4.64	4.66
Production price	5.61	5.04	4.83	4.69	4.62	4.59	4.58	4.58	4.59	4.61
Wage rate	6.69	6.19	7.35	7.80	8.11	8.30	8.43	8.51	8.57	8.62
Export competitiveness	-4.63	-3.48	-2.56	-1.90	-1.43	-1.09	-0.84	-0.66	-0.53	-0.43
Import competitiveness	-3.05	-1.87	-1.17	-0.67	-0.34	-0.11	0.04	0.13	0.19	0.23
Employment										
Firms employment	2.98	1.26	1.27	1.16	1.13	1.11	1.10	1.08	1.07	1.05
Total employment	2.98	1.26	1.27	1.16	1.13	1.11	1.10	1.08	1.07	1.05
Unemployment rate (*)	10.22	10.80	11.21	11.57	11.84	12.05	12.21	12.34	12.43	12.51
Others										
Margins rate (*)	91.78	91.80	91.82	91.83	91.84	91.84	91.85	91.85	91.85	91.85
Profits rate (*)	42.97	42.60	42.89	43.23	43.61	43.97	44.31	44.61	44.88	45.12

with a much more stable evolution.

4.5 The Shocks

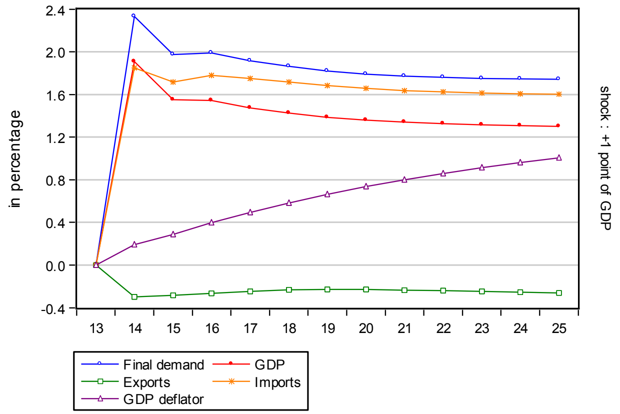
We produce the same shocks as for South Africa, with the following results.

On the whole, the results are rather similar. The main differences are:

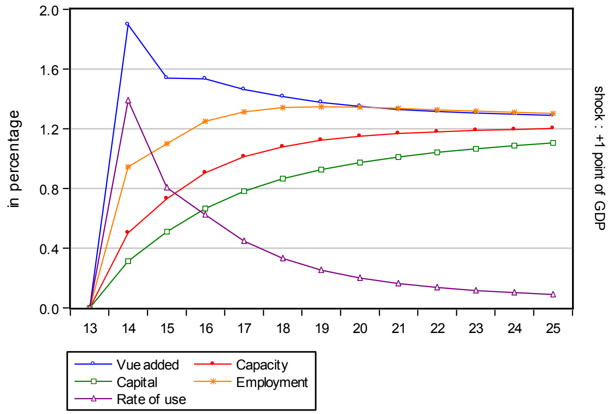
- The shock on quotas applied to Senegal takes more time in taking full effect. This is due to the low short term elasticity of exports to world demand.
- The shock on local tariffs never brings growth. This is due to the low impact of competitiveness on exports, and the limited decrease of local inflation.

4.5.1 An Increase Government Demand

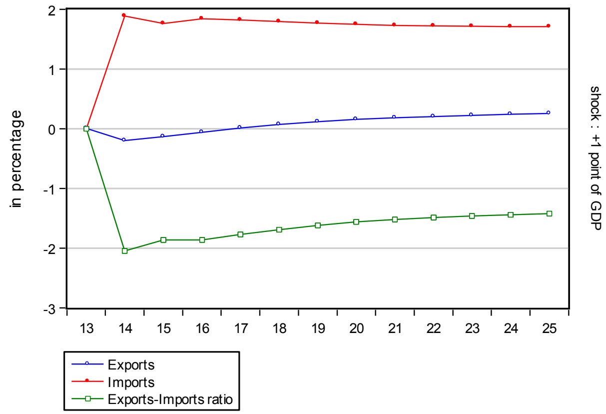
Government demand : Graph no 1 : The supply - demand equilibrium



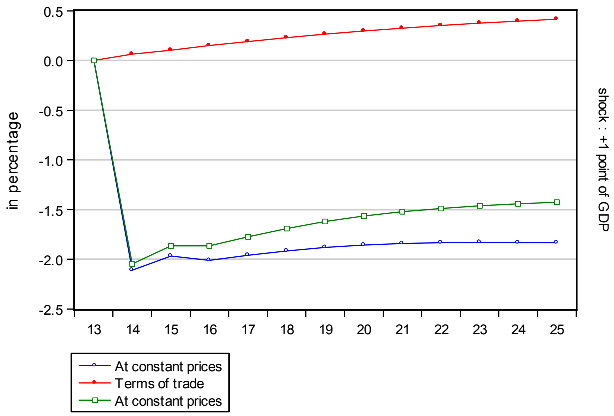
Government demand : Graph no 2 : The production elements



Government demand : Graph no 4 : External trade at current prices



Government demand :Graph no 5 : The exports - import ratios



Government demand : Graph no 5 : The prices

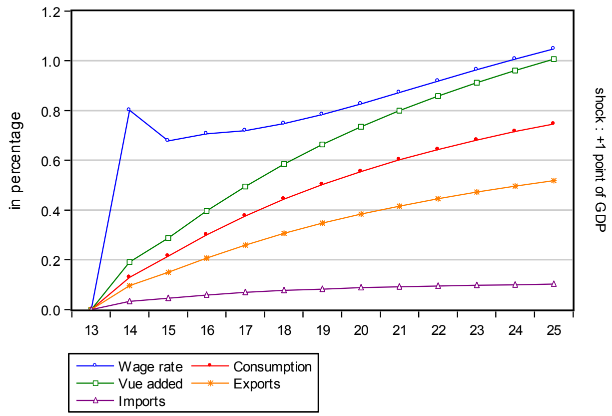
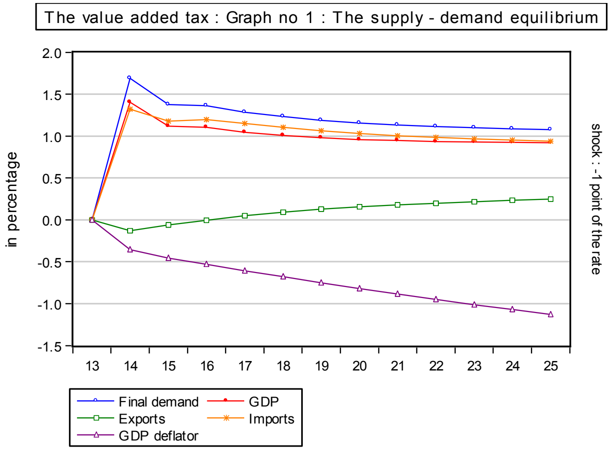
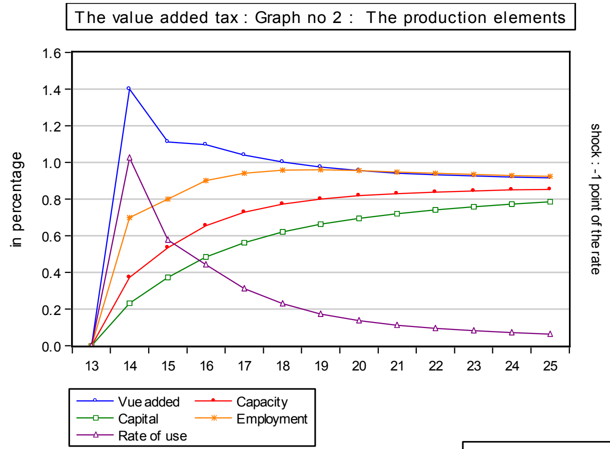


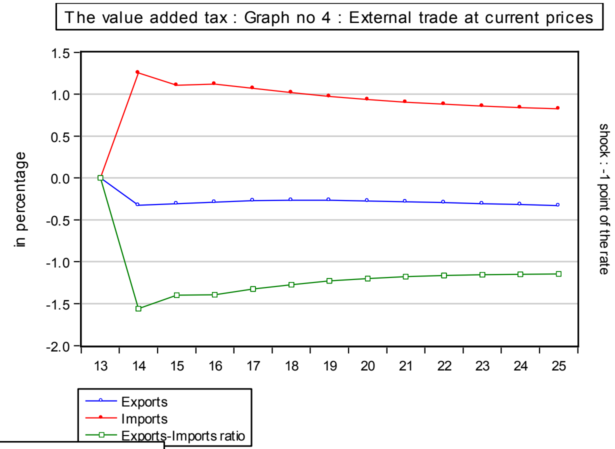
Table 12. Senegal: A Shock on Government Demand

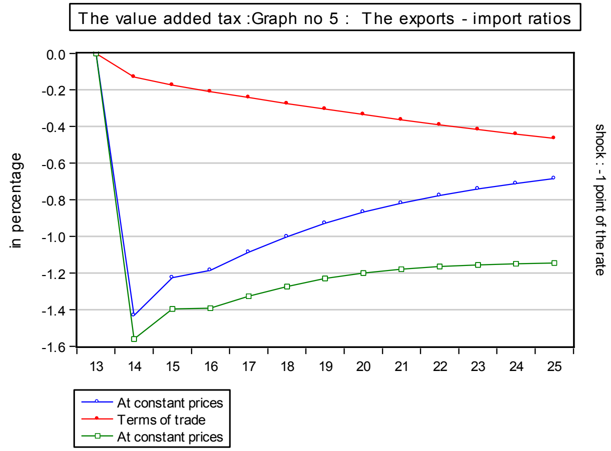
	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Real equlibrium										
GDP	1.89	1.54	1.53	1.46	1.41	1.37	1.34	1.32	1.30	1.29
Final demand	2.31	1.96	1.98	1.90	1.85	1.80	1.77	1.75	1.74	1.73
Productive Investment	3.43	2.50	2.24	1.91	1.68	1.52	1.41	1.35	1.30	1.28
Total Investment	6.45	5.80	5.64	5.42	5.27	5.16	5.10	5.05	5.03	5.01
Household Consumption	1.04	1.02	1.06	1.03	1.00	0.96	0.94	0.92	0.90	0.89
Imports	1.84	1.71	1.77	1.73	1.70	1.66	1.64	1.62	1.60	1.59
Exports	-0.30	-0.29	-0.27	-0.25	-0.24	-0.23	-0.24	-0.24	-0.25	-0.25
Export-import ratio	-2.11	-1.96	-2.00	-1.95	-1.90	-1.87	-1.84	-1.83	-1.82	-1.81
Productive capacity	0.50	0.73	0.90	1.00	1.07	1.11	1.13	1.15	1.16	1.17
Rate of use (*)	1.37	0.80	0.62	0.44	0.32	0.24	0.19	0.15	0.13	0.11
Deflators										
GDP	0.19	0.29	0.41	0.51	0.60	0.68	0.76	0.83	0.89	0.94
Exports	0.10	0.15	0.21	0.27	0.32	0.36	0.40	0.43	0.46	0.49
Imports	0.03	0.05	0.06	0.07	0.08	0.09	0.09	0.09	0.10	0.10
Consumer price index	0.13	0.22	0.31	0.39	0.46	0.52	0.58	0.62	0.67	0.71
Wage rate	0.80	0.69	0.72	0.74	0.77	0.81	0.86	0.91	0.96	1.00
Export competitiveness	0.10	0.15	0.21	0.27	0.32	0.36	0.40	0.43	0.46	0.49
Import competitiveness	-0.13	-0.21	-0.30	-0.37	-0.45	-0.51	-0.57	-0.62	-0.67	-0.71
Employment										
Firms employment	0.94	1.09	1.24	1.30	1.33	1.33	1.33	1.32	1.30	1.29
Total employment	0.94	1.09	1.24	1.30	1.33	1.33	1.33	1.32	1.30	1.29
Unemployment rate (*)	-0.61	-0.72	-0.81	-0.85	-0.87	-0.87	-0.87	-0.86	-0.86	-0.85
Others										
Margins rate (*)	0.01	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Profits rate (*)	0.84	0.56	0.47	0.39	0.32	0.28	0.25	0.23	0.22	0.21
(*) in growth rates										

4.5.2 An Increase in VAT









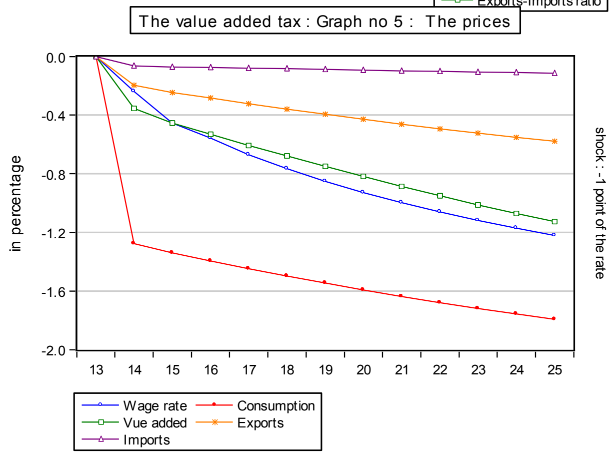
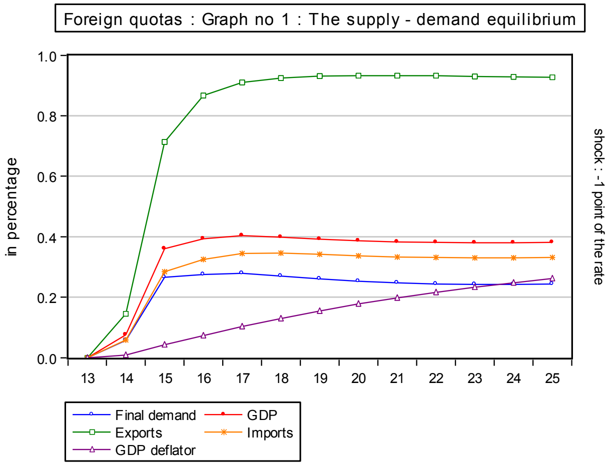
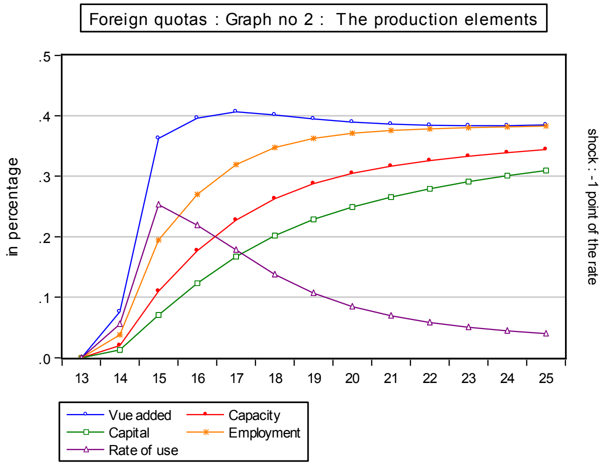


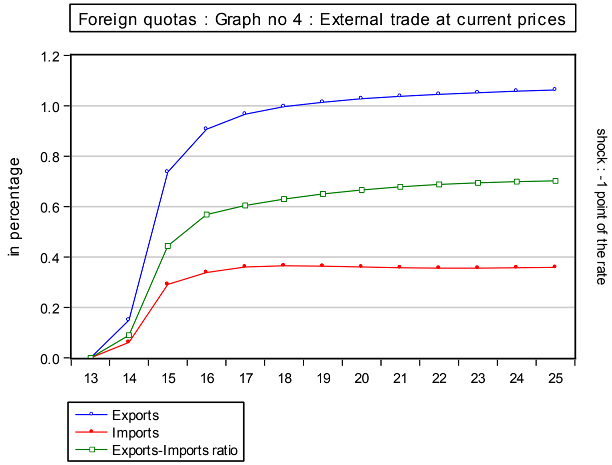
Table 13: Senegal: A Shock On the VAT Rate

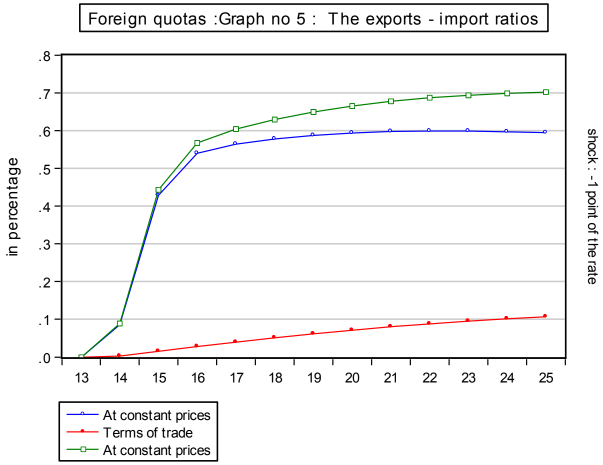
	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Real equlibrium										
GDP	1.40	1.11	1.09	1.03	0.99	0.96	0.94	0.92	0.91	0.90
Final demand	1.68	1.37	1.35	1.27	1.21	1.17	1.13	1.11	1.09	1.07
Productive Investment	2.55	1.79	1.59	1.34	1.17	1.06	0.98	0.94	0.91	0.89
Total Investment	1.82	1.28	1.13	0.95	0.83	0.75	0.70	0.67	0.65	0.63
Household Consumption	1.75	1.68	1.67	1.62	1.57	1.53	1.49	1.46	1.44	1.42
Imports	1.32	1.17	1.19	1.14	1.09	1.05	1.01	0.98	0.96	0.94
Exports	-0.13	-0.06	-0.01	0.05	0.09	0.12	0.15	0.17	0.19	0.21
Export-import ratio	-1.43	-1.22	-1.18	-1.08	-0.99	-0.92	-0.86	-0.81	-0.76	-0.73
Productive capacity	0.37	0.53	0.65	0.72	0.76	0.79	0.80	0.81	0.82	0.82
Rate of use (*)	1.02	0.57	0.43	0.30	0.22	0.16	0.13	0.10	0.09	0.07
Deflators										
GDP	-1.53	-1.63	-1.71	-1.78	-1.85	-1.93	-2.00	-2.06	-2.13	-2.19
Exports	-0.19	-0.24	-0.28	-0.31	-0.34	-0.38	-0.41	-0.44	-0.47	-0.49
Imports	-0.06	-0.07	-0.07	-0.08	-0.08	-0.08	-0.09	-0.09	-0.10	-0.10
Consumer price index	-1.27	-1.33	-1.38	-1.42	-1.47	-1.51	-1.55	-1.59	-1.63	-1.67
Wage rate	-0.23	-0.44	-0.54	-0.65	-0.74	-0.82	-0.89	-0.96	-1.02	-1.07
Export competitiveness	-0.19	-0.24	-0.28	-0.31	-0.34	-0.37	-0.41	-0.44	-0.47	-0.49
Import competitiveness	0.26	0.33	0.39	0.44	0.49	0.54	0.59	0.64	0.68	0.73
Employment										
Firms employment	0.69	0.79	0.89	0.93	0.94	0.94	0.94	0.93	0.92	0.91
Total employment	0.69	0.79	0.89	0.93	0.94	0.94	0.94	0.93	0.92	0.91
Unemployment rate (*)	-0.46	-0.52	-0.59	-0.61	-0.62	-0.62	-0.61	-0.61	-0.60	-0.60
Others										
Margins rate (*)	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Profits rate (*)	1.01	0.76	0.68	0.60	0.54	0.49	0.45	0.42	0.39	0.37
(*) in growth rates										

4.5.3 An Increase in Quotas Applied to Local Exports









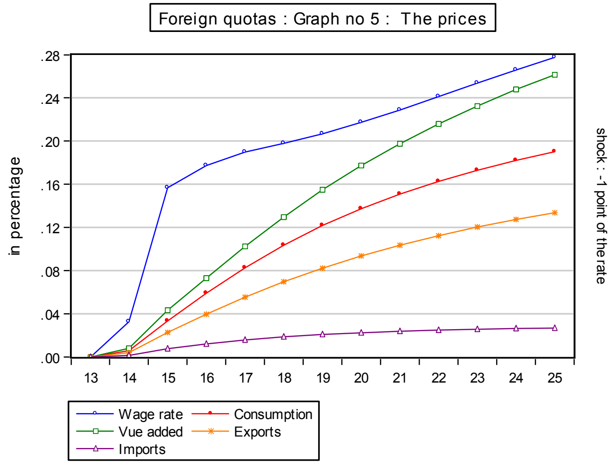
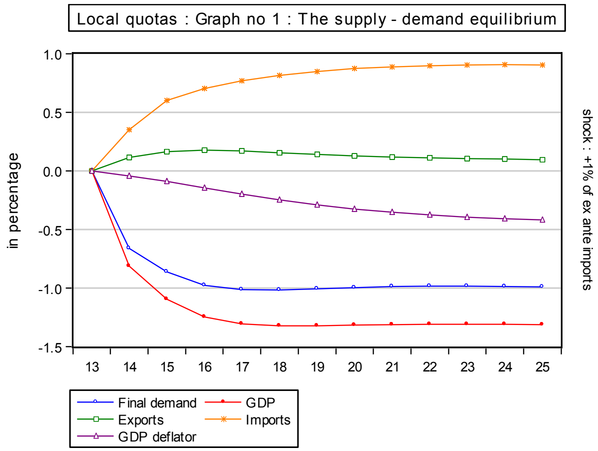
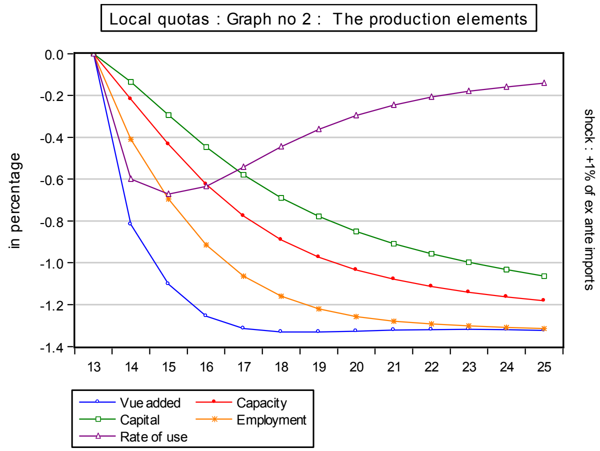


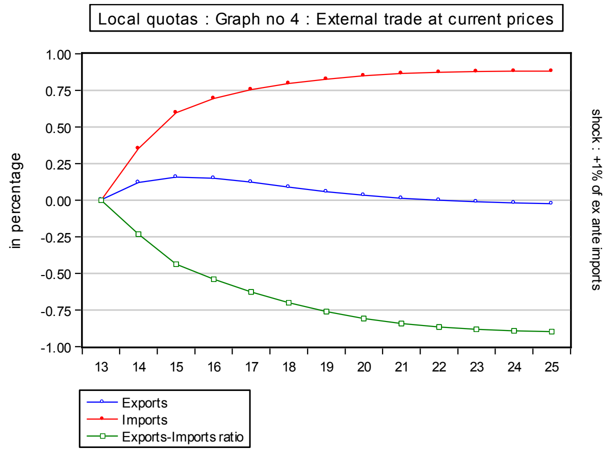
Table 14: Senegal: A Shock on Foreign Quotas

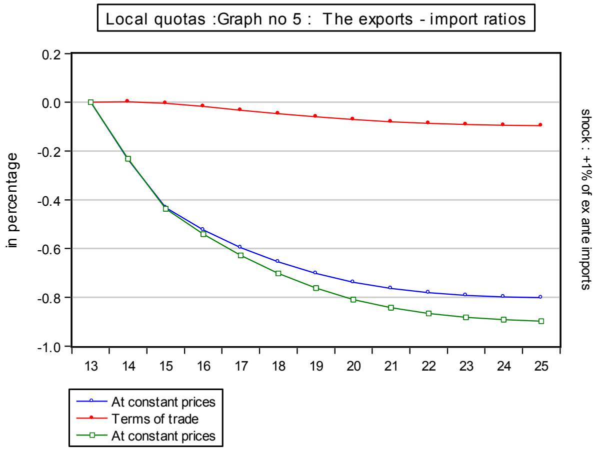
	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Real equlibrium										
GDP	0.08	0.36	0.40	0.41	0.40	0.39	0.39	0.39	0.39	0.38
Final demand	0.06	0.27	0.28	0.28	0.27	0.26	0.26	0.25	0.25	0.25
Productive Investment	0.15	0.66	0.66	0.61	0.55	0.50	0.46	0.43	0.42	0.41
Total Investment	0.11	0.47	0.47	0.44	0.39	0.35	0.33	0.31	0.30	0.29
Household Consumption	0.05	0.21	0.25	0.28	0.28	0.28	0.28	0.27	0.27	0.27
Imports	0.06	0.29	0.33	0.35	0.35	0.34	0.34	0.34	0.33	0.33
Exports	0.16	0.72	0.87	0.91	0.93	0.93	0.93	0.93	0.93	0.93
Export-import ratio	0.09	0.44	0.54	0.56	0.58	0.58	0.59	0.59	0.59	0.59
Productive capacity	0.02	0.11	0.18	0.23	0.27	0.29	0.31	0.32	0.33	0.34
Rate of use (*)	0.06	0.25	0.22	0.18	0.14	0.11	0.09	0.07	0.06	0.05
Deflators										
GDP	0.01	0.04	0.07	0.10	0.13	0.16	0.18	0.20	0.22	0.24
Exports	0.00	0.02	0.04	0.06	0.07	0.08	0.09	0.10	0.11	0.12
Imports	0.00	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.03
Consumer price index	0.01	0.03	0.06	0.08	0.10	0.12	0.14	0.15	0.16	0.17
Wage rate	0.04	0.16	0.18	0.19	0.20	0.21	0.22	0.23	0.25	0.26
Export competitiveness	0.00	0.02	0.04	0.06	0.07	0.08	0.09	0.10	0.11	0.12
Import competitiveness	-0.01	-0.03	-0.05	-0.08	-0.10	-0.12	-0.13	-0.15	-0.16	-0.17
Employment										
Firms employment	0.04	0.20	0.27	0.32	0.35	0.37	0.37	0.38	0.38	0.38
Total employment	0.04	0.20	0.27	0.32	0.35	0.37	0.37	0.38	0.38	0.38
Unemployment rate (*)	-0.03	-0.13	-0.18	-0.21	-0.23	-0.24	-0.25	-0.25	-0.25	-0.25
Others										
Margins rate (*)	0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Profits rate (*)	0.04	0.16	0.15	0.13	0.11	0.10	0.09	0.08	0.08	0.07
(*) in growth rates										

4.5.4 An Increase in Quotas Applied to Local Imports









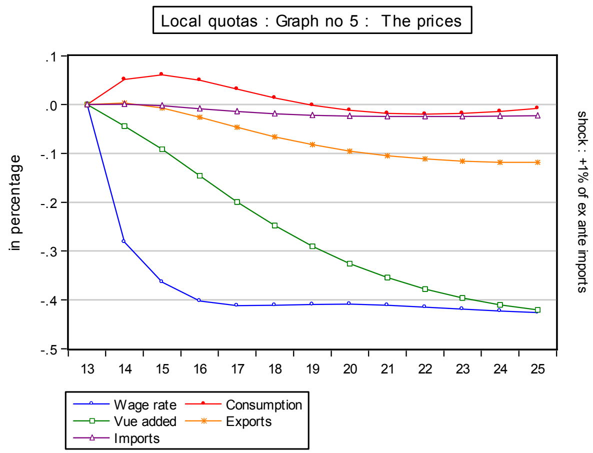
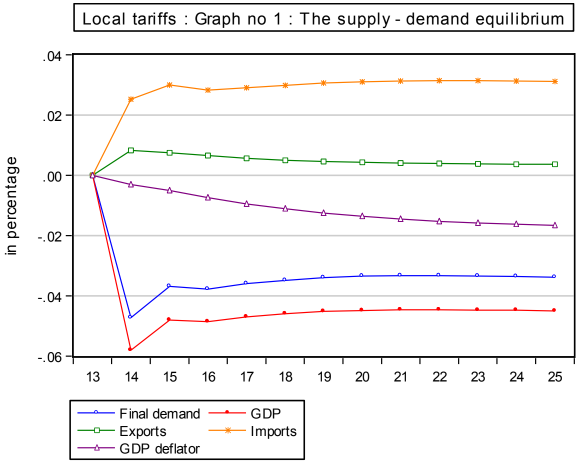
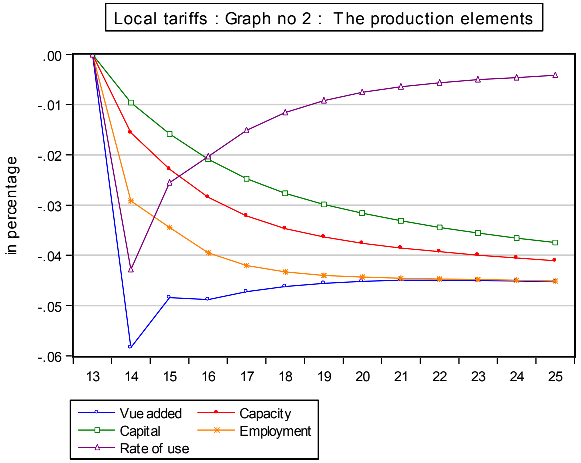


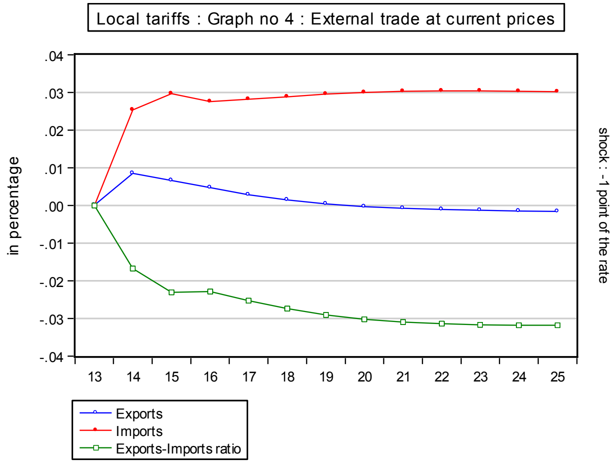
Table 15: Senegal: A shock on local quotas

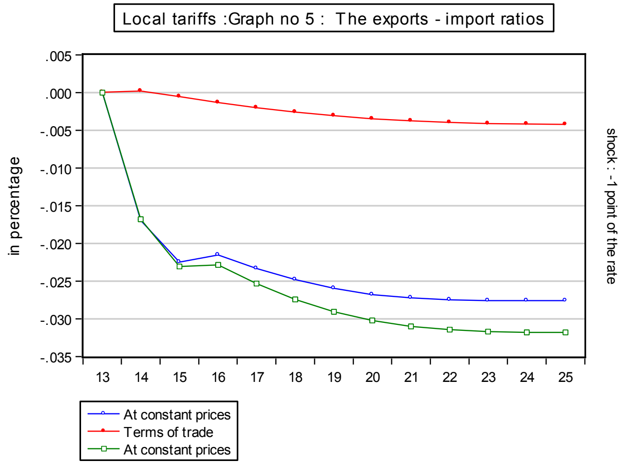
	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Real equlibrium										
GDP	-0.81	-1.12	-1.28	-1.35	-1.37	-1.37	-1.37	-1.37	-1.37	-1.37
Final demand	-0.67	-0.88	-1.01	-1.06	-1.07	-1.06	-1.05	-1.05	-1.04	-1.04
Productive Investment	-1.50	-1.93	-2.06	-1.99	-1.87	-1.74	-1.63	-1.55	-1.50	-1.46
Total Investment	-1.07	-1.37	-1.46	-1.42	-1.33	-1.23	-1.16	-1.10	-1.06	-1.04
Household Consumption	-0.53	-0.83	-1.01	-1.11	-1.17	-1.19	-1.20	-1.21	-1.22	-1.23
Imports	0.35	0.59	0.70	0.76	0.81	0.84	0.87	0.88	0.89	0.90
Exports	0.12	0.17	0.18	0.17	0.16	0.14	0.12	0.11	0.10	0.10
Export-import ratio	-0.23	-0.42	-0.51	-0.59	-0.65	-0.70	-0.74	-0.76	-0.78	-0.79
Productive capacity	-0.22	-0.44	-0.64	-0.80	-0.92	-1.00	-1.07	-1.12	-1.16	-1.19
Rate of use (*)	-0.60	-0.68	-0.66	-0.56	-0.47	-0.38	-0.31	-0.26	-0.22	-0.19
Deflators										
GDP	-0.04	-0.08	-0.14	-0.19	-0.24	-0.28	-0.31	-0.34	-0.36	-0.37
Exports	0.01	-0.00	-0.02	-0.04	-0.05	-0.07	-0.08	-0.08	-0.09	-0.09
Imports	0.00	-0.00	-0.01	-0.01	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
Consumer price index	0.06	0.08	0.07	0.06	0.05	0.04	0.04	0.04	0.04	0.05
Wage rate	-0.28	-0.36	-0.41	-0.42	-0.41	-0.41	-0.41	-0.41	-0.41	-0.41
Export competitiveness	0.01	-0.00	-0.02	-0.03	-0.05	-0.07	-0.08	-0.08	-0.09	-0.09
Import competitiveness	-0.01	0.00	0.02	0.05	0.07	0.09	0.11	0.12	0.13	0.13
Employment										
Firms employment	-0.41	-0.71	-0.93	-1.09	-1.20	-1.26	-1.30	-1.33	-1.34	-1.35
Total employment	-0.41	-0.71	-0.93	-1.09	-1.20	-1.26	-1.30	-1.33	-1.34	-1.35
Unemployment rate (*)	0.27	0.47	0.62	0.72	0.79	0.83	0.86	0.88	0.89	0.89
Others										
Margins rate (*)	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00
Profits rate (*)	-0.40	-0.50	-0.52	-0.50	-0.46	-0.43	-0.40	-0.37	-0.36	-0.34

4.5.5 A Decrease in the Local Tariffs Rate









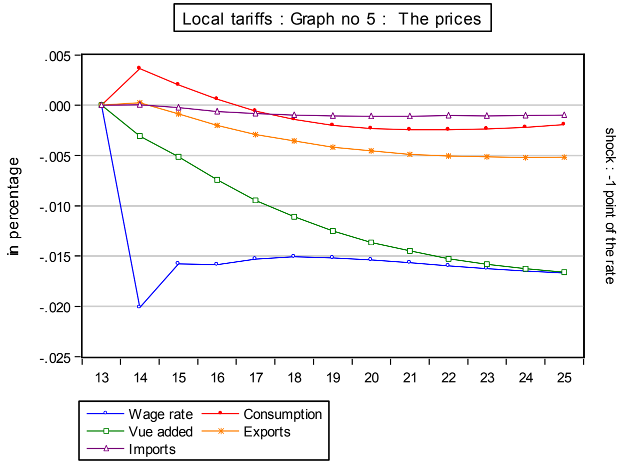


Table 16. Senegal: A Shock on Local Tariffs

	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Real equlibrium										
GDP	-0.06	-0.05	-0.05	-0.05	-0.05	-0.05	-0.04	-0.04	-0.04	-0.04
Final demand	-0.05	-0.04	-0.04	-0.04	-0.04	-0.03	-0.03	-0.03	-0.03	-0.03
Productive Investment	-0.11	-0.08	-0.07	-0.06	-0.06	-0.05	-0.05	-0.05	-0.05	-0.05
Total Investment	-0.08	-0.06	-0.05	-0.04	-0.04	-0.04	-0.04	-0.03	-0.03	-0.03
Household Consumption	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04
Imports	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Exports	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Export-import ratio	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.03	-0.03	-0.03	-0.03
Productive capacity	-0.02	-0.02	-0.03	-0.03	-0.03	-0.04	-0.04	-0.04	-0.04	-0.04
Rate of use (*)	-0.04	-0.03	-0.02	-0.02	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
Deflators										
GDP	-0.00	-0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
Exports	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Imports	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Consumer price index	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Wage rate	-0.02	-0.02	-0.02	-0.02	-0.01	-0.01	-0.01	-0.02	-0.02	-0.02
Export competitiveness	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Import competitiveness	-0.86	-0.86	-0.86	-0.86	-0.86	-0.85	-0.85	-0.85	-0.85	-0.85
Employment										
Firms employment	-0.03	-0.03	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04
Total employment	-0.03	-0.03	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04
Unemployment rate (*)	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Others										
Margins rate (*)	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Profits rate (*)	-0.03	-0.02	-0.02	-0.02	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
(*) in growth rates										

4.5.6 A Devaluation

Table 17. Senegal: A devaluation of the local currency

Real equilibrium										
GDP	0.05	0.06	0.05	0.04	0.04	0.03	0.03	0.02	0.02	0.02
Final demand	0.01	0.01	0.00	-0.01	-0.02	-0.02	-0.03	-0.03	-0.03	-0.03
Productive Investment	0.10	0.10	0.08	0.06	0.04	0.03	0.02	0.01	0.01	0.01
Total Investment	0.07	0.07	0.06	0.04	0.03	0.02	0.01	0.01	0.01	0.01
Household Consumption	-0.02	-0.01	-0.02	-0.03	-0.03	-0.04	-0.04	-0.04	-0.05	-0.04
Imports	-0.01	-0.01	-0.02	-0.02	-0.03	-0.04	-0.04	-0.04	-0.04	-0.04
Exports	0.17	0.20	0.20	0.20	0.19	0.18	0.17	0.16	0.15	0.14
Export-import ratio	0.18	0.21	0.22	0.22	0.22	0.21	0.21	0.20	0.19	0.18
Productive capacity	0.01	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.02
Rate of use (*)	0.04	0.03	0.02	0.01	0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Deflators										
GDP	0.08	0.12	0.17	0.21	0.25	0.29	0.32	0.36	0.39	0.43
Exports	0.48	0.52	0.55	0.58	0.60	0.63	0.65	0.67	0.69	0.71
Imports	0.63	0.72	0.79	0.84	0.88	0.90	0.92	0.93	0.94	0.95
Consumer price index	0.19	0.26	0.32	0.37	0.42	0.46	0.49	0.52	0.55	0.58
Wage rate	0.14	0.18	0.23	0.26	0.30	0.33	0.37	0.40	0.43	0.46
Export competitiveness	-0.51	-0.48	-0.45	-0.42	-0.39	-0.37	-0.35	-0.33	-0.31	-0.29
Import competitiveness	0.49	0.53	0.55	0.55	0.54	0.52	0.50	0.48	0.46	0.44
Employment										
Firms employment	0.03	0.04	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.02
Total employment	0.03	0.04	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.02
Unemployment rate (*)	-0.02	-0.03	-0.03	-0.03	-0.03	-0.02	-0.02	-0.02	-0.02	-0.01
Others										
Margins rate (*)	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Profits rate (*)	-0.03	-0.04	-0.05	-0.06	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07
(*) in growth rates										

5. Bibliography

We have limited this work to elementary econometrics notions, and we have used only macroeconomics to illustrate some methods. We give you now a list of more specialized, or less basic, reference works.

Econometrics and Statistics

Baltagi B.H. 2008. Econometrics. Springer.

Baltagi, B.H. 2007. Econometric Analysis of Panel Data (4^{th} ed.). Wiley.

Banerjee, A. et al. 1993. Co-Integration, Error Correction, and the Econometric Analysis of Non-Stationary Data (Advanced Texts in Econometrics). Oxford University Press.

Carter Hill, R., W.E. Griffiths, and G. C. Lim. 2011. Principles of Econometrics (4^{th} ed.). John Wiley & Sons.

Charemza, W., D. Deadman. 1997. New Directions in Econometric Practice: General to Specific Modelling, Cointegration, and Vector Auto regression. Edward Edgar Pub.

Clements, P., and D. F Hendry. 1998. Forecasting Economic Time Series. Cambridge University Press.

Corbae, D., M. B. Stinchcombe, J. Zeman. 2009. An Introduction to Mathematical Analysis for Economic Theory and Econometrics. Princeton University Press.

Diebold, F.X. 2000. Elements of Forecasting. South Western, 2ème Ed.

Enders, W. 2004. Applied Econometric Time Series, Wiley Series in Probability and Statistics (2^{nd} ed.). Wiley.

Gomez, V., A. Maravall. 1996. Programs TRAMO (Time series Regression with ARIMA noise, Missing observations, and Outliers) and SEATS (Signal Extraction in ARIMA Time Series). Instructions for the User. *Working Paper 9628*. Banco de España.

Gourieroux, C., A. Monfort, G. Gallo. (translator) 1997. Time Series and Dynamic Models (Themes in Modern Econometrics). Cambridge University Press.

Gourieroux, C., A. Monfort, Q. Vuong. (translator). 1995. Statistics and Econometric Models (Themes in modern econometrics). Cambridge University Press.

Greene, W.H. 2011. Econometric Analysis (7^{th} ed.). Pearson.

Greene, W.H. 2011. Econometric Analysis (7^{th} ed.). Prentice Hall.

Gukarati, D. N. 2010. Basic Econometrics (5^{th} ed.). McGraw-Hill Companies (first published 1995).

Hamilton, J D. 1994. Times Series Analysis. Princeton University Press.

Harvey, G. C. 1990. The Econometric Analysis of Time Series. MIT Press: Cambridge, MA.

Hayashi, F. 2000. Econometrics. Princeton University Press.

Heij, C., and P.H. Franses. 2004. Econometric Methods with Applications in Business and Economics. Oxford University Press, USA.

Hendry, D. F. 1995. Dynamic Econometrics. Oxford University Press.

Johnston, J. 1990. Econometric Methods Irwin/McGraw-Hill.

Juselius, K. 2006. The Cointegrated VAR Model: Methodology and Applications. Oxford University Press.

Kennedy, P. 2008. A Guide to Econometrics (6th ed.). Blackwell Publishers.

Ladiray, D., and B. Quenneville. 2001. Seasonal adjustment with the X-11 Method. Springer.

Lim, G.C. 2011. Study Guide for Using Eviews for Principles of Econometrics. Academic Internet Publishers.

Lütkepohl, H., M. Krätzig. 2004. Applied Time Series Econometrics. Cambridge.

Mackinnon, J. G. 2003. Econometric Theory and Methods. Oxford University Press, USA.

Maddala, G. S. 1994. Econometric Methods and Applications (Economists of the Twentieth Century). Edward Elgar Pub.

Ngurah Agung, I. G. 2009. Time Series Data Analysis Using Eviews. John Wiley & Sons.

Pindyck, R.S., and D. L. Rubinfeld. 2000. Econometric Models and Economic Forecasts.

Rao B. 2007. Cointegration: For the Applied Economist (2nd ed.). St. Martins Press.

Lucas, R. E., Jr. 1976. Econometric Policy Evaluation: A Critique.' Carnegie-Rochester Conference Series on Public Policy' 1, pp. 19–46.

Stock, J. H., M. W. Watson. 2010. Introduction to Econometrics, (Addison-Wesley Series in Economics) (3rd ed.). Wesley Publishing Company.

Thompson, J. R. 2011. Empirical Model Building: Data, Models, and Reality. John Wiley & Sons.

William E., R. Carter, Hill, and G.C. Lim. 2011. Using EViews for Principles of Econometrics (4th ed.). John Wiley & Sons.

Woolridge J. M. 2005. Introductory Econometrics: A Modern Approach. Thomson South-Western.

X-12-Arima Reference Manual. 2011. Version 0.3.Time Series Research Staff, Statistical Research Division, U.S. Census Bureau, WWW: http://www.census.gov/srd/www/x12a/

Macroeconomics

Agenor, P.-R., P. J. Montiel. 2008. Development Macroeconomics (3rd ed.). Princeton University Press.

Aghion, P., P. W. Howitt. 2008. The Economics of Growth, MIT Press.

Arnold, R. A. 2013. Macroeconomics (11th ed.). South Western Educational Publishing (first published December 1995).

Barro, R. J. 1997. Macroeconomics (5th ed.). MIT Press: Cambridge, MA.

Barro, R.J. and X. Sala-I-Martin. 2003. Economic Growth (2nd ed.). Mc Graw-Hill, New York.

Behrman, J., & Srinivasan, T.N. (Eds.) 1995. Handbook of Development Economics (vol. III). Elsevier.

Blanchard, O. 2012. Macroeconomics (6th ed.). Prentice Hall.

Blanchard, O. J., and S. Fisher. 1989. Lectures on Macroeconomics MIT Press: Cambridge, MA.

Burda, M., and C. Wyplosz C. 2009. Macroeconomics, A European text (5th ed.). Oxford University Press.

Chenery, H., and T. N. Srinivasan. 1989. Handbook of Development Economics, North Holland, Amsterdam.

Gali, J. 2008. Monetary Policy, Inflation, and the Business Cycle. Princeton University Press.

Grossman, G. M., and K. Rogoff. 1995. Handbook of International Economics. (vol. 3). Elsevier, North Holland.

Lequiller, F., and D. Blades. 2006. Understanding National Accounts. OECD Publishing, available at: http://www.oecd.org/dataoecd/37/12/38451313.pdf.

Mankiw, N. G. 2012. Macroeconomics (8th ed.). Wh Freeman (first published 1991).

Mankiw, N. G. Principles of Macroeconomics (6th ed.). Thomson South-Western.

Muet, P A. 1990. Théories et modèles de la macroéconomie.. Economica.

Obstfeld, M., and K. S. Rogoff. 1996. Foundations of International Macroeconomics. MIT Press: Cambridge, MA.

Ray, D. 1998. Development Economics. Princeton University Press.

Romer, D. 2011. Advanced Macroeconomics (4th ed.). Mcgraw Hill Higher Education.

Sachs, J. D., and Bono. (Foreword) 2006. The End of Poverty. Penguin Books.

Stachurski, J. 2009. Economic Dynamics: Theory and Computation. MIT Press.

Taylor, J.B. 1993. "Discretion versus Policy Rules in Practice". Carnegie-Rochester Conference Series on Public Policy, 39, pp.195-21.

Taylor, J.B., and M. Woodford. 1999. Handbook of Macroeconomics. Elsevier, North Holland.

Woodford, M 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press.

Models and Modelling

Amman, H. M., D. A. Kendrick., and J. Rust. (eds.). 1996. Handbook of Computational Economics (vol. 1). North-Holland, Amsterdam.

Amman, H. M., B. Rustem, and A. B. Whinston. 1996. Computational Approaches to Economic Problems. (vol. 6). *Advances in Computational Economics*. Kluwer Academic Publishing.

Allen, C., and S. Hall (eds.). 1997. Macroeconomic Modelling In A Changing World: Towards A Common Approach. John Wiley & Sons.

Artus, P., M. Deleau, and P. Malgrange. 1986. Modélisation Macroéconomique.Paris: Economica.

Bodkin, R. G., L. R. Klein, and K. Marwah. 1991. A History of Macroeconometric Model Building. Edward Elgar.

Brillet JL. 1994. Modélisation Econométrique: Principes et Techniques. Paris: Economica.

Canova, F. 2007. Methods for Applied Macroeconomic Research. Princeton University Press.

Carnot, N., and B. Tissot. 2002. La Prévision Economique. Paris: Economica.

Christ C.F. 1994 "The Cowles Commission Contributions to Econometrics at Chicago: 1939-1955". *Journal of Economic Literature. Vol. 32*.

Dejong, D. N., and C. Dave. 2007. Structural Macroeconometrics. Princeton University Press.

Dervis, K., J. de Melo., and S. Robinson. 1982. General Equilirium Models for Development Policy. World Bank: Washington D.C.

Fair, R. C. 1994. Testing Macroeconometric Models. Harvard University Press.

Fair, R.C. 2004. Estimating How the Macroeconomy Works. Harvard University Press.

Fair, R.C. 2014. Reflections on Macroeconometric Modeling Ray C. Fair Revised January 2014.

Fair, R.C. http://fairmodel.econ.yale.edu/

Fisher, P. 1992. Rational Expectations in Macroeconomic Models (Advanced Studies in Theoretical & Applied Econometrics (vol. 26). Kluwer.

Fromm G., L. R. Klein, and E. Kuh. 1965. The Brookings-S.S.R.C. Quarterly Econometric Model of the United States: Model Properties. Brookings Institution, 1965 -22 pages.

Fouquet D. et al. 1978. DMS, Modèle Dynamique Multi Sectoriel. Collections de l'INSEE no 64-65.

Gantmacher, F. R.1959. Theory of Matrices. AMS Chelsea publishing.

Gilli M. 1996. Computational Economic Systems: Models, Methods & Econometrics (Advances in Computational Economics) (vol. 5). Kluwer Academic Publishing.

Gilli, M., and E. Rossier. 1980. Understanding and Solving Complex Systems. Automatica. (vol. 17, pages 647-652).

Hall, S. G., and S. G. B. Henry. 1989. Macroeconomic Modelling. North Holland.

Intriligator M. D., R. G. Bodkin, and C. Hsiao. 1995. Econometric Models, Techniques and Applications. Prentice Hall Press.

Bardsen, G., et al. 2005. The Econometrics of Macroeconomic Modelling. Oxford University Press.

Evans, M. K., and L. R. Klein. 1967. The Wharton Econometric Forecasting Model, Economics Research Unit. University of Pennsylvania.

Klein L. R., and A. S. Goldberger. 1955. An Econometric Model of the United States, 1929–52. North-Holland.

Klein, L. R., and G. Fromm. 1975. The Brookings Model: Perspective and Recent Developments. Elsevier.

Klein, L. R. 1982. Econometric Models as Guides for Decision Making. New York Free Press.

Klein, L. R. 1991. Comparative Performance of U.S. Econometric Models. Oxford University Press

Kydland, F., E. and Prescott. 1977. Rules Rather Than Discretion: The Inconsistency of Optimal Plans. Journal of Political Economy, 85, 473-490.

Laffargue, J. P. 1990. Résolution D'un Modèle Économique Avec Anticipations Rationnelles. Annales d'Economie et de Statistiques no 17.

Laxton. D. 1998. Multimod Mark III: The Core Dynamic and Steady-State Models. IMF.

Malgrange, P., P. A. Muet. 1985. Contemporary Macroeconomic Modelling. Blackwell Publishers.

Mcadam P., A. J. Hughes Hallett. 1999. Advances in Computational Economics (vol. 12). Analyses in Macroeconomic Modelling. Springer.

Mesange Model: http://www.insee.fr/fr/publications-et-services/docs_doc_travail/G2010-17.pdf.

MZE Model: http://www.insee.fr/fr/ffc/docs_ffc/ES451J.pdf.

Nepomiaschy, P., and A. Ravelli. 1978. Adapted Methods for Solving and Optimizing Quasi--Triangular Econometric Models, Annals of Economic and Social Measurement.

Pindyck, R., and Rubinfeld D. 1997. Econometric Models and Economic Forecasts. Cambridge University Press.

Pontragin, L.S. et al. 1962. The Mathematical Theory of Optimal Processes (Russian), English translation. Interscience.

Sadoulet, E., and A. De Janvry. 1995. Quantitative Development Analysis. John Hopkins University Press.

Sbodorne, A.et al. 2010. 'Policy Analysis using DSGE Models: an Introduction'. Federal Reserve Bank of New York Economic Policy Review, 16 (2).

Shoven, J., and J. Whalley. 1992. Applying General Equilibrium (Cambridge Surveys of Economic Literature). Cambridge University Press.

Taylor, L. (ed.). 1990. Socially Relevant Policy Analysis, Structuralist Computable General Equilibrium Models for the Developing World. MIT Press.

Tinbergen, J. 1939. Business Cycles in the United States. 1919–1932. Geneva.

Tinbergen, J. 1969. Business Cycles in the United States. 1919–1932. New York.

Tinbergen, J. 1956. Economic Policy: Principles and Design. North Holland.

Wallis K. F. (ed.). 1988. Models of the U.K. Economy: A Fourth Review by the Esrc Macroeconomic Modelling Bureau. Oxford University Press, USA.

Whitney, J.D. 1994. A Course in Macroeconomic Modelling and Forecasting. Harvester Wheatsheaf.